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ON IRREDUCIBLE MAPS

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The notion of irreducible map was introduced by M. Auslander and I. Reiten in [3] and plays an important role in the representation theory of artin algebras.

We recall that an artin ring Λ is said to be an artin algebra if its center C is an artin ring and Λ is finitely generated left Λ-module. Now choose a complete set $P_1, \ldots, P_s$ of representatives of the isomorphism classes of indecomposable projectives in $\mathrm{mod}(\Lambda)$, we will denote by $\mathrm{pr}\ \Lambda$ the full subcategory of $\mathrm{mod}\ \Lambda$ whose objects are $P_1, \ldots, P_s$. A map $g\colon X \longrightarrow Y$ in $\mathrm{mod}(\Lambda)$ is said to be irreducible if g is neither a split monomorphism nor a split epimorphism and for any commutative diagram

$$\begin{tikzcd} X \arrow[rr, "g"] \arrow[dr, "f"'] & & Y \\ & Z \arrow[ur, "h"'] & \end{tikzcd}$$

f is a splittable monomorphism or h is a splittable epimorphism.

We study irreducible maps in $\mathrm{mod}(\Lambda)$ by using properties of the Jacobson radical of $\mathrm{mod}(\Lambda)$. We recall that the Jacobson radical of $\mathrm{mod}(\Lambda)$ is the subfunctor rad of the two variable functor $\mathrm{Hom}\colon (\mathrm{mod}(\Lambda))^{\mathrm{op}} \times \mathrm{mod}(\Lambda) \longrightarrow \mathrm{Ab}$ defined by

$$\begin{aligned}\mathrm{rad}(X, Y) &= \{f \in \mathrm{Hom}(X, Y) \mid 1 - gf \text{ is invertible for every } g \in \mathrm{Hom}(Y, X)\} \\ &= \{f \in \mathrm{Hom}(X, Y) \mid 1 - fh \text{ is invertible for any } h \in \mathrm{Hom}(X, Y)\}.\end{aligned}$$

It is easy to prove that if X and Y are indecomposables, then $\mathrm{rad}(X, Y)$ consists of all nonisomorphisms, from X to Y.

We can prove the following result:

PROPOSITION 1. *Let C and D be indecomposables in* $\mathrm{mod}(\Lambda)$. *Then*

(a) *A map $f\colon C \longrightarrow$ is irreducible iff $f \in \mathrm{rad}(C, D)$ and $f \notin \mathrm{rad}^2(C, D)$, where $\mathrm{rad}^2(C, D)$ consists of all maps of the form $t_1 t_2$ with $t_2 \in \mathrm{rad}(C, X)$ and $t_1 \in \mathrm{rad}(X, D)$.*

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