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A TOPOLOGICAL RESOLUTION THEOREM

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We prove a topological analogue of the resolution theorem for algebraic varieties [**H**]. We show that every compact P.L. manifold M admits a framed stratification (every stratum has a product neighborhood) such that after a sequence of topological blow ups performed along the closed smooth strata we get a compact smooth manifold $\widetilde{M}$ ($\partial\widetilde{M} = \varnothing$ if $\partial M = \varnothing$) and a degree one map (with $Z/2$ coefficients) $\pi: \widetilde{M} \longrightarrow M$. The map π is a P.L. homeomorphism in the complement of a union of smooth submanifolds of the form $N_i \times W_i$, such that π collapses $N_i \times W_i$ to N_i in some order. This structure can be used to show that every compact P.L. manifold is P.L. homeomorphic to a real algebraic variety [**AK**]. This also gives a nice way of defining differential forms on P.L. manifolds by pushing down the relative forms from the smooth resolution spaces.

Define an A_0-structure on a P.L. manifold to be a smooth structure, and call such manifold an A_0-manifold. Inductively define an A_k-structure on a P.L. manifold M to be a decomposition

$$M = M_0 \cup_\phi \coprod_{i=1}^{r} N_i \times \operatorname{cone}(\Sigma_i)$$

for some r, where M_0 is an A_{k-1}-manifold with boundary; each Σ_i is a boundary of a compact A_{k-1}-manifold and is P.L. homeomorphic to a P.L. sphere; and N_i are smooth manifolds. Finally $\phi = \{\phi_i\}$ are maps describing the identification (as stratified sets) $\phi_i: N_i \times \Sigma_i \longrightarrow \partial M_0$ where the union is taken. We say M has an A-structure if it has an A_k-structure for some k.

To describe the blowing up process, let M be an A_k-manifold. Then $M = M_0 \cup \coprod_i N_i \times \operatorname{cone}(\Sigma_i)$ and we can choose compact A_{k-1}-manifolds W_i with $\partial W_i = \Sigma_i$. Construct the obvious A_{k-1}-manifold $\widetilde{M}_{k-1} = M_0 \cup \coprod_i N_i \times W_i$. There is the obvious P.L. map $\pi: \widetilde{M}_{k-1} \longrightarrow M$ which is the identity on M_0 and collapses each $N_i \times W_i$ onto N_i. We can iterate this process to get a resolution sequence

$$\widetilde{M} = \widetilde{M}_0 \xrightarrow{\pi} \widetilde{M}_1 \xrightarrow{\pi} \cdots \xrightarrow{\pi} \widetilde{M}_{k-1} \xrightarrow{\pi} M.$$

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