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RESEARCH ANNOUNCEMENTS

NONLINEAR SIMILARITY OF MATRICES

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Works of Poincaré [7], de Rham [8], [9],[2] Reidemeister, Kuiper and Robbin [6], Sullivan, and Schultz [11] showed that in a large number of cases, various types of nonlinear similarity of real matrices with eigenvalues all of modulus one,[3] implies linear similarity of these matrices. In [4] we gave the first examples of matrices with eigenvalues of modulus one which are *nonlinearly similar* but, as they have different traces, are not linearly similar. Therefore the classification of such matrices up to nonlinear similarity is much different from what had been conjectured on the basis of the earlier results. This paper begins the systematic *classification up to nonlinear similarity of matrices with eigenvalues of modulus one* (and group representations) *which are not linearly similar.* This could be called the *topological canonical form problem for matrices.* For a large class of matrices (and group representations) we completely solve this problem.

Two real entried invertible $n \times n$ matrices A and B are *nonlinearly* or *topologically similar* if there is a homeomorphism f: $R^n \longrightarrow R^n$ with[4] $f(0) = 0$ and $fAf^{-1} = B$; here A and B are regarded as (linear) homeomorphisms of R^n.

For matrices with eigenvalues of modulus 1, and without roots of unity as eigenvalues, or with all eigenvalues which are roots of unity being sth roots of unity with $s = 1, 2, 3, 4$, or 6, Kuiper and Robbin showed that nonlinear similarity implies linear similarity. This is also known for matrices all of whose eigenvalues are primitive sth roots of unity, for a fixed s; see [6]. Dennis Sullivan, and Reinhard Schultz [11] showed nonlinear similarity implies similarity for matrices whose eigenvalues are p^s or $2p^s$ roots of unity, p an odd prime.

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[2]For a detailed proof of de Rham's result and extensions to the P. L. case see Rothenberg [10].

[3]The classification of all matrices up to nonlinear similarity reduces to the case of matrices with eigenvalues of modulus 1, see Kuiper and Robbin [6], cf. [1].

[4]If the condition $f(0) = 0$ is dropped the equivalence classes of topologically similar matrices are not changed.

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