not cover in detail; he discusses many applications of Bessel polynomials; calls attention to some unsolved problems about Bessel polynomials; and outlines their history in a preliminary chapter (from which I have borrowed most of the historical remarks in this review).

It is too much to hope that the appearance of this book will prevent the Bessel polynomials from being reinvented, but it will be useful to anyone who comes across them, or one of their variants, and is resourceful enough to find it. Perhaps eventually someone will organize the literature of orthogonal polynomials in inverse form, listing desirable properties and typical problems, and indicating which polynomials have the properties or help solve the problems. Until the arrival of that millennial day, treatises like this one are all we can reasonably expect, and we should be duly grateful to Grosswald for making the Bessel polynomials more accessible.

REFERENCES

1. A. Erdélyi et al., *Higher transcendental functions*, vol. 2, McGraw-Hill, New York, Toronto, London, 1953.

2. H. L. Krall and O. Frink, *A new class of orthogonal polynomials: the Bessel polynomials*, Trans. Amer. Math. Soc. **65** (1949), 100–115.

3. G. Szegö, *Orthogonal polynomials*, American Mathematical Society, New York, 1939.

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BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 1, Number 5, September 1979

0002-9904/79/0000-0415/$01.50

The finite element method for elliptic problems, by Philippe G. Ciarlet, North-Holland, Amsterdam, New York, Oxford, 1978, xvii + 530 pp., $56.95.

There is a wide variety of numerical techniques, particularly for the solution of partial differential equations, that go under the heading of finite element methods. The most elementary version of these methods occurs in the context of the Poisson equation where they typically are a special case of the classical variational methods developed by Galerkin, Rayleigh, and Ritz. The latter are based on the Dirichlet Principle which asserts that the solution of the boundary value problem uniquely minimizes a quadratic functional, normally called the energy functional, over a certain class U of functions. The classical idea is to obtain approximations by minimizing the energy functional over a finite-dimensional subspace of U [1], [2]. What distinguishes finite element methods in this context is the particular choice of the finite-dimensional subspace that is used in the approximation. In particular, finite element methods are typically based on spaces of piecewise polynomial functions associated with a simplical decomposition of the region.

There is not general agreement concerning the originator of these ideas although most numerical analysts quote either Courant [3] or Synge [4], both of whom had the basic ideas concerning the elementary mechanics of the method. Later in the mid 1960s engineers independently started an intensive development of the method [5], and several successful applications to large and complicated problems that were originally thought to be intractable generated almost overnight popularity in engineering circles.