To this reviewer, the time seems ripe for experts in lattice theory to reconsider the challenging question asked G. D. Birkhoff in 1933 [3, p. 7]. After listening attentively to my earnest explanation of some of their basic properties, he asked: "What can be proved using lattices that cannot be proved without them?" Not only was this question the main theme of the 1938 symposium [1] at which lattice theory was first given publicity by the American Mathematical Society, but its stimulus still strongly pervaded the much deeper 1960 symposium [2] on the same subject.

Instead, Professor Grätzer draws a careful line between "lattice theory proper and its allied fields", and avoids discussing results which "belong to universal algebra and not to lattice theory". This partly neutralizes his important but brief comment at the beginning of Chapter V, that "of the four characterizations given, three apply to arbitrary equational classes of universal algebras". Although it may be most efficient for the product of Ph.D. theses to subdivide mathematics up into neat, self-contained branches, the vitality of mathematics depends in the long run on a widespread familiarity with interconnections between these branches, and even on ideas coming from other areas of science.

Nevertheless, for those who already appreciate lattice theory, or who are curious about its techniques and intriguing internal problems, Professor Grätzer's lucid new book provides a most valuable guide to many recent developments. Even a cursory reading should provide those few who may still believe that lattice theory is superficial or naive, with convincing evidence of its technical depth and sophistication.

REFERENCES
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3. *Trends in lattice theory*, J. C. Abbott (ed.), van Nostrand, New York, 1970.

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Crystallographic groups of four-dimensional space, by Harold Brown, Rolf Bülow, Joachim Neubüser, Hans Wondratschek and Hans Zassenhaus, Wiley, New York, 1978, xiv + 443 pp., $38.90.

In Euclidean n-space, a *crystallographic space group* is a discrete group of isometries which contains, as a subgroup, the group generated by n independent translations. For this subgroup, which is abstractly C_∞^n (the free Abelian group with n generators), the orbit of any point is a *lattice*, which may alternatively be described as an infinite discrete set of points whose set of position vectors is closed under subtraction. The word 'crystallographic' is used because the positions of atoms in a crystal are well represented by lattices (with $n = 3$) or by sets of superposed lattices. For instance, the *cubic* lattice, of points whose Cartesian coordinates are integers, describes the