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THE QUEER DIFFERENTIAL EQUATIONS FOR ADIABATIC COMPRESSION OF PLASMA

BY GUDMUNDUR VIGFÚSSON[1]

This announcement presents some results on the "Queer Differential Equations" (QDE) of adiabatically evolving plasma equilibria. These are nonlinear differential-functional equations of the form $\Delta\psi = F(V, \psi, \psi', \psi'')$, where $V = V(\psi)$ is the volume (area) inside the levelsets $\psi(\mathbf{r}) = \psi$, a constant, and the derivatives on the right hand side are with respect to the dependent variable V, e.g., $\psi'(V)$.

We describe properties of microcanonical averages and their derivatives and a simple example of the nonlinear problem. An existence and uniqueness theorem is given for the associated linearized problem, which is also a functional-differential equation. Finally we will mention an isoperimetric problem related to the geometry of QDE's.

A few of these results are in [1] and will be further expounded in [2]–[4]. For the initial development of this problem and the relevant physics see [5]–[7] and the references therein. For a related problem, see also [8], [9].

Averages. S is a simple domain in the plane and $z = \psi(\mathbf{r})$, $\mathbf{r} \in \bar{S}$, a surface such that $\psi \in C^{n,\alpha}(\bar{S})$, $n \geqslant 2$, $0 \leqslant \alpha < 1$. Assume that the level lines $\psi(\mathbf{r}) = \psi$ are simple and $\nabla\psi = \mathbf{0}$ at only one point $\mathbf{r}_0 \in S$, $\psi(\mathbf{r}_0) = \psi_0$. Let ∂S be the level line $\psi(\mathbf{r}) = \psi_1$ ($> \psi_0$). $V = V(\psi)$ is defined to be the area inside the curve $\psi(\mathbf{r}) = \psi$ and $\psi(V)$ is the inverse of $V(\psi)$.

$$V'(\psi) = \oint_{\psi} |\nabla\psi|^{-1}\, ds$$

is continuous in the interval $I = (\psi_0, \psi_1]$.

The function $V(\mathbf{r}) = V(\psi(\mathbf{r}))$ is useful; it is C^1 in $\bar{S}\backslash\mathbf{r}_0$.

THEOREM 1. *Let ψ be as described above and assume that $\mathbf{r}_0$ is an elliptic critical point. Then $V(\mathbf{r}) = V(\psi(\mathbf{r}))$ is also in $C^{n,\alpha}$ and $\mathbf{r}_0$ is an elliptic critical point for $V = V(\mathbf{r})$.*

Consider $g \in C^{m,\alpha}(\bar{S})$ and define on I,

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