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ON THE COMPLETE INTEGRABILITY OF NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS

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The concept of complete integrability for nonlinear Hamiltonian systems of finite dimension $2N$ is generally based on a theorem of Liouville (cf. [**1**, p. 271]), that requires the Hamiltonian system to possess N independent first integrals in involution. Recently this notion has been extended to infinite-dimensional Hamiltonian systems by a number of authors (for example, Faddeev and Zacharov, Gardner, Lax, McKean, Novikov, Gelfand and Dikii, among others) who have shown that certain nonlinear partial differential equations in two dimensions are integrable in this sense of Liouville provided one lets $N \longrightarrow \infty$. In particular the celebrated Korteweg-de Vries

$$u_t + uu_x + u_{xxx} = 0 \tag{1}$$

is completely integrable in this sense.

However, this notion of complete integrability seems to be of limited value for treating nonlinear partial differential equations in more than two variables. Moreover, the study of the perturbations of a system completely integrable in this sense of Liouville generally requires radically new methods, since the first integrals (whose existence is intrinsic to the Liouville approach) are generally destroyed.

In this article we define a new type of complete integrability for nonlinear elliptic boundary value problems (in fact for nonlinear mappings between Banach spaces), and show, by example, that an explicit nonlinear Dirichlet problem (π_n) defined on an arbitrary bounded domain $\Omega \subset \mathbf{R}^n$ with dimension *n arbitrary* is completely integrable in our sense. Moreover, our methods of study are sufficiently flexible to yield significant results for a C^1 perturbation $\widetilde{\pi}_n$ of π_n.

1. The notion of complete integrability. Let A denote a given smooth (say C^{k+1}) mapping between two real Banach spaces X_1, X_2. Then we say A is *globally C^k equivalent* to a mapping B between X_3 and X_4 if there are C^k diffeomorphisms α and β such that the following diagram commutes

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