25. M. Sato, T. Kawai, and M. Kashiwara, *Microfunctions and pseudo differential equations*, Lecture Notes in Math., vol. 287, Springer-Verlag, New York, 1973, pp. 265–529.

26. M. Schechter, Spectra of partial differential operators, North-Holland, Amsterdam, 1971.

27. E. Stein, Singular integral operators and differentiability properties of functions, Princeton Univ. Press, Princeton, N. J., 1970.

28. E. Stein, and G. Weiss, Introduction to Fourier analysis on Euclidean spaces, Princeton Univ. Press, Princeton, N. J., 1971.

29. K. Yosida, Functional analysis, Springer-Verlag, New York, 1965.

MICHAEL E. TAYLOR

BULLETIN (New Series) OF THE AMERICAN MATHEMATICAL SOCIETY Volume 1, Number 4, July 1979 © 1979 American Mathematical Society 0002-9904/79/0000-0311/\$01.50

Information and exponential families in statistical theory, by O. Barndorff-Nielsen, Wiley, New York, 1978, ix + 238 pp., \$33.00.

If we take n samples of a random variable which is normally distributed with unknown mean  $\mu$  and known variance  $\sigma^2$ , then the sample mean  $\overline{X}$  is normally distributed with mean  $\mu$  and variance  $\sigma^2/n$ . Thus the probability that  $\sqrt{n} (\overline{X} - \mu) / \sigma$  lies between -1.96 and 1.96 is, from the standard normal table, equal to .95 and hence it seems that the probability that  $\mu$  lies between  $\overline{X} - 1.96\sigma/\sqrt{n}$  and  $\overline{X} + 1.96\sigma/\sqrt{n}$  is also .95. The latter statement, which seems to the naive reader to be equivalent to its immediate predecessor, makes no sense since the fixed number  $\mu$  either lies in the given interval (a 95% confidence interval) or it doesn't and most elementary books on statistics take some pains to explain this fact. What the statement really means is that if we use this method over and over again to infer that  $\mu$  is in the given interval we will be right about 95% of the time. Unfortunately this limit of frequency concept coincides with most people's intuitive idea of probability and very likely was used that way earlier in the same book. The same situation obtains in the more realistic situation of an unknown  $\sigma$  except that the t-distribution is used instead of the normal and many other statistical problems lead to this sort of impasse.

The apparent duality here; each choice of  $\mu$  determining a distribution of  $\overline{X}$  and each sample mean  $\overline{X}$  seeming to determine a distribution of  $\mu$ ; has lead to a great deal of heated dispute among statisticians, who tend to be rather feisty anyhow. The first great storm center seems to have been the fertile brain of R. A. Fisher. Fisher's idea of fiducial inference is not much mentioned anymore but the debate goes on as the book under review illustrates.

The book is divided into three parts the first of which is concerned, among other things, with the above mentioned duality. In fact, x serves as the probability variable while  $\omega$  is reserved for the parameter. This emphasizes the duality perhaps at the expense of occasionally confusing the over-conditioned probabilist. Barnard's notion of l.o.d.s. functions is introduced but not extensively developed. The likelihood function and Barndorff-Nielsen's somewhat controversial plausibility function seem to be the only well-known examples. Much of this part is devoted to the, more or less, dual notions of sufficiency and ancillarity. In fact four different definitions are given of each,