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Infinite loop spaces, by J. F. Adams, Ann. of Math. Studies no. 90, Princeton Univ. Press, Princeton, N. J., 1978, x + 214 pp., $14.00 (cloth), $5.50 (paper).

Algebraic topology is a very young and restless subject. Many of its most active areas did not exist a decade ago, or existed only in embryonic form. Examples include the theory of localizations and completions, rational homotopy theory, the study of finite H-spaces, exploitation of the Brown-Peterson spectrum and other new techniques in the calculation of stable homotopy groups, algebraic K-theory and the homotopy theory of categories, exploitation of techniques from algebraic geometry, and infinite loop space theory and its applications. Similarly, many techniques and constructions that are already accepted by workers in the field as standard and elementary are equally new. One thinks of localizations, completions, the classifying spaces of monoids and categories, the geometric transfer, the plus construction, etc. Not one of these things is so much as mentioned in even the most recent and advanced texts in the subject (and if this sounds like a rebuke to their authors, why so be it).

If this is true of standard and elementary parts of the subject, then it is hardly surprising that the deeper machinery relevant to the more sophisticated new areas is virtually inaccessible without direct contact with practitioners. The goal of Adams' book is to "convey the basic ideas of the subject in a way as nearly painless as I can make it". By "the subject" he means infinite loop space theory. But in fact he has succeeded in giving the basic ideas not just of this specialty but of much of modern algebraic topology, including capsule introductions to many of the topics mentioned above. I urge anybody teaching algebraic topology on any level and anybody working or thinking of working in the subject to read this book. One or two patches might be a little hard going, particularly in Chapter 6 (and the reader is given fair warning), but for the most part the book provides some of the most delightful and illuminating exposition to be found, not just in topology, but in mathematics. It is written with style and wit, and reads like a novel (in places, as on pp. 112 and 144, like a roman a clef, although the characters in the drama are usually identified even when being chided). The truth is, whatever we may say, that we invent mathematics because it is fun. Seldom has the spirit of the enterprise been so successfully captured in print. Would that treatments in a similar vein were available in other supposedly impenetrable abstract areas of mathematics.

Nevertheless, of course, the emphasis is on infinite loop space theory, and some of my colleagues don't much see the fun in that. There are various areas of mathematics that are widely regarded with suspicion and dislike because of the quantity of pure abstraction involved. There are also various areas that are widely regarded with suspicion and dislike because of the quantity of grubby calculation involved. Infinite loop space theory runs simultaneously to both extremes and so has something to offend everyone.