BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 1, Number 4, July 1979

0002-9904/79/0000-0301/$02.50

BOOK REVIEWS

Anti-invariant submanifolds, by Kentaro Yano and Masahiro Kon, Lecture Notes in Pure and Applied Mathematics, vol. 21, Marcel Dekker, Inc., New York, 1976, vii + 183 pp., $19.75.

General introduction. This article is divided into two parts: Part I quickly recalls the basic concepts of differential geometry, including the notions of differentiable manifolds and Kählerian and Sasakian manifolds, while Part II is the review of the Yano-Kon book. (Some readers, familiar with differential geometry, may wish to skip Part I. For their benefit I have repeated the definitions of Kählerian and Sasakian manifolds at the start of Part II.)

Part I. Review of differential geometry. A *topological n-manifold* is a metrizable topological space M which locally looks like $\mathbf{R}^n$, in the following sense: each point $p \in M$ has a neighborhood U homeomorphic to some open set W in $\mathbf{R}^n$. If $x = (x_1, \ldots, x_n)$: $U \to W \subset \mathbf{R}^n$ is such a homeomorphism, then the pair (U, x) is called a *coordinate chart*. Two such charts (U, x) and (V, y) are *smoothly related* if either (a) $U \cap V = \varnothing$, or (b) $U \cap V \neq \varnothing$ and the maps $x \circ y^{-1}$: $y(U \cap V) \to x(U \cap V)$ and $y \circ x^{-1}$: $x(U \cap V) \to y(U \cap V)$ (defined on the open subsets $y(U \cap V)$ and $x(U \cap V)$ in $\mathbf{R}^n$) are smooth (i.e., of class C^∞). A *differentiable n-manifold* is a topological n-manifold M on which a class F of coordinate charts has been singled out. The class F must satisfy (a) every $p \in M$ is in some chart of F and (b) if (U, x) and (V, y) are charts in F, then they are smoothly related.

If M is a differentiable n-manifold, one can "do calculus on M". For example, one can introduce the notion of differentiable functions on M: a function f: $M \to \mathbf{R}$ is of class C^k if for each chart $(U, x) \in F$ the function $f \circ x^{-1}$: $x(U) \to \mathbf{R}$ is of class C^k on the open set $x(U)$ in $\mathbf{R}^n$. Similarly, by considering compositions of the form $x \circ \gamma$, one can define a notion of differentiable maps γ from (say) an interval in $\mathbf{R}$ to M.

Associated with each point p of a differentiable manifold M is an n-dimensional real vector space, the *tangent space* $T_p M$. The fact that M admits such "linear approximations" is the central feature of the theory of differentiable manifolds; in particular, it explains the constant use of linear and multilinear algebra so characteristic of this theory. The most intuitive description of the tangent space is this: if γ: $(a, b) \to M$ is a smooth curve at p (i.e., a smooth map into M of an interval $a < t < b$ containing $t = 0$ such that $\gamma(0) = p$) and if $(U, x) \in F$ is a chart containing p, then we can associate to γ a vector in $\mathbf{R}^n$, namely $v_\gamma = d((x \circ \gamma)(t))/dt|_{t=0}$; we say that two such curves at p, γ and σ, are equivalent if $v_\gamma = v_\sigma$ and we denote the equivalence class of γ by $[\gamma]$; then an element of T_pM is simply such an equivalence class and the vector space structure on T_pM is that induced by the bijection $[\gamma] \mapsto v_\gamma$ of T_pM with $\mathbf{R}^n$. (This construction does not depend on the particular chart at p that we happen to use.)

Analogously, we define a *complex n-manifold* to be a metrizable topological space M which is covered by a prescribed family F of "holomorphically