BULLETIN (New Series) OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 1, Number 3, May 1979

A HIGHER DIMENSION GENERALIZATION OF THE SINE-GORDON EQUATION AND ITS BÄCKLUND TRANSFORMATION

BY KETI TENENBLAT[1] AND CHUU-LIAN TERNG[2]

The classical Bäcklund theorem ([1], [4], [5]) studies the transformation of hyperbolic (i.e. constant negative curvature) surfaces in R^3 by realizing them as focal surfaces of pseudo-spherical line congruences. The integrability theorem says that one can construct a family of new hyperbolic surfaces in R^3 from a given one. Bianchi showed how to construct algebraically another family of hyperbolic surfaces from this family.

It is well known that there is a correspondence betwen solutions of the Sine-Gordon equation

SGE $$\frac{\partial^2 \phi}{\partial x^2} - \frac{\partial^2 \phi}{\partial t^2} = \sin \phi$$

and hyperbolic surfaces in R^3 ([1], [4], [5]). Therefore Bäcklund's theorem provides a method for generating new solutions of SGE from a given one, and Bianchi's permutability theorem [5] enables one to construct more solutions by an algebraic formula. This technique has recently received much attention in the studies of soliton solutions of SGE [2] and has been used successfully in the study of solitons of other nonlinear equations of evolution in one space dimension. But generalizations to more space variables has been less successful.

A natural generalization would be to find a transformation theory for hyperbolic (i.e. constant negative sectional curvature) submanifolds in Euclidean space. É. Cartan [3] showed that hyperbolic n-manifolds locally immerse in R^{2n-1}, but not in R^{2n-2}. Moreover, [3] he proved the existence of "line of curvature coordinates", in which all components of the second fundamental form are diagonalized. J. D. Moore [6] improved this result and we have:

THEOREM 1 (É. CARTAN). *Suppose M is a hyperbolic n-submanifold of R^{2n-1}. Then locally M can be parametrized by its lines of curvature so that*

Received by the editors December 4, 1978.
AMS (MOS) subject classifications (1970). Primary 53B25; Secondary 35L60.
[1]Work done under partial support of CNP_9.
[2]Work done under partial support of NSF grant MCS 76-01692.

0002-9904/79/0000-0222/$02.25