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AUTOMORPHIC FORMS AND SINGULARITIES OF COMPLEX SURFACES

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DEDICATED TO MY WIFE, BOBBIE

Suppose G is a finitely generated fuchsian group of the first kind. Let $A(k)$ be the vector space of entire automorphic forms of weight k and

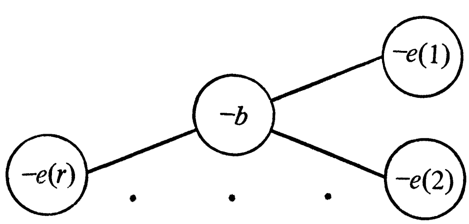
$$A(G) = \bigoplus_{k \geq 0} A(k)$$

the graded ring of automorphic forms. Now G acts on the upper half plane H_+ in the usual way. This action has a 'canonical' extension to $H_+ \times \mathbf{C}^*$ via

$$g(z, t) = \left(g(z), \frac{dg}{dz} t\right).$$

PROPOSITION 1. *$A(G)$ is a graded algebra of finite type. The algebraic set $V = \mathrm{Spec}(A(G))$ is a surface with $\mathbf{C}^*$-action. There is a Zariski open $\mathbf{C}^*$-invariant subset of V which is isomorphic to $(H_+ \times \mathbf{C}^*)/G$.*

We thus can use the theory of surfaces with $\mathbf{C}^*$-action to study the structure of the ring of automorphic forms. Now let us suppose that G is a fuchsian group with signature $\langle g; s; e(1), \ldots, e(r)\rangle$ and $V = \mathrm{Spec}(A(G))$. By [7] the singularity of V at (0) has a canonical equivariant resolution. The graph of the resolution is star shaped, of the form



where $b = 2g - 2 + r + s$.

A first step in understanding the structure of these rings is to find the minimal number of generators, n. In [9] we classified all groups with $n \leq 3$. The techniques there are all elementary. The results here are more general since we use

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