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TRAVERSAL TIMES OF MARKOV PROCESSES

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There are two main results we wish to state which are proved in a series of papers starting with [1]. The first is that whenever a particle which is undergoing motion governed by a strong Markov process with stationary transition probabilities and without holding points traces out a fixed trajectory segment, it must trace it out in a fixed length of time depending only on the transition probabilities and on the trajectory segment. This theorem has as an immediate corollary the fact that a continuous Markov process on the real line which always moves in the same direction must do so deterministically. The second main result, an application of the first, is that if two processes have the same hitting probabilities and the second is a strong Markov process with stationary transition probabilities, then the first can be transformed into the second by a nonanticipating change of the time scale, without having to enlarge the σ-fields. This is a sharpening of the result given in [2] for two arbitrary processes with the same hitting probabilities and no holding points, but only in the case that the second process is stong Markov.

The first result may be stated formally as follows. There is a set $\Omega_1 \subset \Omega$, with $P_\mu(\Omega_1) = 1$ for all initial distributions μ, such that if $H = \{X_t(\omega)\colon \omega \in \Omega_1\}$ then H is closed under shifts, and H is consistently traced. That H is closed under shifts means that if $X_t(\omega) \in H$, then $X_{t+s}(\omega) \in H$ for all $s \geqslant 0$. That H is consistenly traced means that if $X_t(\omega)$ and $X_t(\omega')$ are in H and trace out the same trajectory segment, there they must do so at the same rate: if $X_t(\omega)$, $a \leqslant t \leqslant b$ and $X_t(\omega')$, $c \leqslant t \leqslant d$ determine the same trajectory segment, then $b - a = d - c$ and $X_{a+s}(\omega) = X_{c+s}(\omega')$, $0 \leqslant s \leqslant b - a = d - c$.

As is customary in probability theory, *path* denotes a function of time having values in the state space and *trajectory* denotes the ordered set of points traversed by the path, so that, for example, if ω is fixed, $X_t(\omega)$ and $X_{t^2}(\omega)$ are different paths traversing the same trajectory. To carry out the proof of the first result, let $\tau_1 \leqslant \tau_2$ be two stopping times with τ_2 defined by the post-τ_1 behavior of the process. First, the paths of the Markov process are grouped into equivalence classes consisting of those paths which traverse the same trajectory segment as t ranges over the interval $[\tau_1, \tau_2)$, and then the probability measure of the process is disintegrated with respect to these equivalence classes. Next, it is shown that the time of traversal of the segment is, for almost all

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