

the book is a thorough compilation of what was known up to that time about the Basic Problem for (4).

Since the book was written there has been considerable activity in equations of variable type, especially in the Soviet Union. As one might expect in a newly developing field, many of the results are fragmentary. For example, a spate of papers has dealt with equations with discontinuous coefficients. Other papers have developed properties of solutions of variable equations of order higher than two. Still a third group of results is based on the equations of gas dynamics which are of mixed type when the flow has both supersonic and subsonic regions.

It seems likely that the bits and pieces of all these results will be put together eventually to form a single comprehensive theory for mixed type equations in R^2 similar to the classical theory for the standard elliptic, hyperbolic and parabolic equations. At this time the task appears formidable; the development in three and more dimensions and for equations of order other than two is even more remote.

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The Bochner integral, by Jan Mikusiński, Academic Press, New York, 1978, xii + 233 pp.

In the 1930's a great effort was made to develop the basic theory of Banach space valued functions of a real variable. The pioneers in this study (Bochner, Dunford, Gelfand, Pettis, Phillips and Richart) developed a number of integrals of varying strengths for a multitude of purposes: oftentimes, the representation of operators on concrete spaces was the object; quite as often a desire just to understand the abstract process of integration was sufficient motivation. Of the integrals developed, one integral, the Bochner integral, emerged as the strongest and, to-date, it is the Bochner integral that has been the most useful.

Curiously, the Bochner integral is the easiest of the vector integrals from yesteryear to develop and the one with the most transparent structure. Indeed, most of the usual results valid for the Lebesgue integral easily adapt to the Bochner setting. One notable exception: the fundamental theorem of calculus for absolutely continuous functions defined on $[0, 1]$. It is simply not the case that an absolutely continuous vector-valued function defined on $[0, 1]$ need be the indefinite Bochner integral of its derivative—at least not unless the vector values are suitable chosen. This pathology is not all together a bad thing. The study of the class of Banach spaces for which the fundamental theorem remains valid has kept a number of mathematicians busy and off the streets for the past five years at least. This class of spaces (whose members answer to the name “Radon-Nikodym”) has come to play an important role in modern Banach space theory especially as it interacts (and it does so quite nicely) with probability theory, harmonic analysis and the infinite dimensional topology. Any book purporting to be about the Bochner