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BOOK REVIEWS

Algebraic geometry I: *Complex projective varieties*, by David Mumford, Grundlehren der Math. Wissenschaften, vol. 221, Springer-Verlag, Berlin, Heidelberg, New York, 1976, x + 186 pp., $14.80.

During the last twenty years, the field of algebraic geometry has undergone a thorough revision and strengthening of its foundations with the introduction of schemes and sheaf cohomology. This period of rapid growth created a significant gap between what could be found in textbooks and the most recent research. These new methods are now having an impact on neighboring fields such as number theory, complex manifolds, topology, and even the nonlinear differential equations of mathematical physics. This creates an even greater need for introductory books which should make algebraic geometry more accessible.

David Mumford has already written several books on various aspects of algebraic geometry [6], [7], [8], [9]. Indeed his "little red book" [6], now out of print, initiated a generation of students to the subject when nothing else was available. More recently a number of introductory books have appeared more or less simultaneously in different parts of the world [1], [2], [4], [5], [10].

In my earlier review [3] of the books by Dieudonné and Shafarevich, I suggested some of the challenges facing the author of an introduction to algebraic geometry: how can he present the complicated technical tools of the new methods and still give enough examples and geometrical intuition to convey what the subject is all about? In the book under review, Mumford neatly sidesteps this difficulty by deferring the technicalities of schemes and cohomology to a second promised volume. To quote from his Introduction, "... the present volume, which is the first of several, introduces only complex projective varieties. But, as a consequence, we can study these effectively with topological and analytic techniques without extensive preliminary work on 'foundations'. My goal is precisely to convey some of the classical geometric ideas and to get 'off the ground': in fact, to get to the 27 lines on the cubic–surely one of the gems hidden in the rag-bag of projective geometry."

I will not attempt here to explain what is algebraic geometry, nor even to summarize what is in the present book. Let me rather try to give the flavor of Mumford's approach by describing three specific topics which he treats.

The first is Chow's theorem which states roughly that the only complex analytic subsets of complex projective space are algebraic varieties. More precisely, this means that if X is a closed subset of $\mathbf{P}^n_{\mathbf{C}}$, and if each point $x \in X$ has a small neighborhood U in $\mathbf{P}^n_{\mathbf{C}}$, such that $X \cap U$ is equal to the set of common zeros of some family of holomorphic functions $f_1, \ldots, f_q$ on U, then there exist homogeneous polynomials $F_1, \ldots, F_s$ in the homogeneous coordinates $z_0, \ldots, z_n$ of the projective space, so that X is exactly the set of common zeros of the F_i in all of $\mathbf{P}^n_{\mathbf{C}}$. The hypotheses state that X is a complex analytic subset of $\mathbf{P}^n_{\mathbf{C}}$, which is a purely local property of X. The conclusion is