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NONCONVEX MINIMIZATION PROBLEMS[1]

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I. The central result. The grandfather of it all is the celebrated 1961 theorem of Bishop and Phelps (see **[7]**, **[8]**) that the set of continuous linear functionals on a Banach space E which attain their maximum on a prescribed closed convex bounded subset $X \subset E$ is norm-dense in E^*. The crux of the proof lies in introducing a certain convex cone in E, associating with it a partial ordering, and applying to the latter a transfinite induction argument (Zorn's lemma). This argument was later used in different settings by Brøndsted and Rockafellar (see **[9]**) and by F. Browder (see **[11]**). The various situations can be adequately summarized in a diagram:

This paper surveys my 1972 result on the variational principle and the various advances and applications which have been registered since. They stretch over a vast field of mathematics, from control theory to global analysis, and it is hoped that every mathematician will find something to enjoy. This should be possible, because the ideas involved are quite simple, and I have tried not to obscure them by too technical or detailed an exposition; the reader will be referred to the original papers for the more peripheral lemmas; received by the editors September 1, 1978.

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