ASYMPTOTIC STATES FOR EQUATIONS OF REACTION AND DIFFUSION

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- 1. Introduction. The term "equations of reaction and diffusion" is usually taken to mean semilinear systems of second order partial differential equations of the form

$$\partial u/\partial t = D\Delta u + f(x, t, u, \nabla u), \qquad u = (u_1, \dots, u_m),$$
 (1.1)

or generalizations made by replacing the Laplace operator by linear or quasilinear elliptic operators. Here the "diffusion matrix" D has nonnegative

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