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THE SIGNATURE OF SMOOTHINGS OF HIGHER DIMENSIONAL SINGULARITIES

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1. **Introduction.** Let f: $(\mathbf{C}^{2n+1}, 0) \longrightarrow (\mathbf{C}, 0)$ be the germ of a complex analytic function with an isolated critical point at the origin. For $\epsilon > 0$ suitably small and δ yet smaller, the space $V' = f^{-1}(\delta) \cap D_\epsilon$ (where D_ϵ denotes the closed disk of radius ϵ about 0) is a real oriented $2n$-manifold with boundary whose diffeomorphism type depends only on f. It has been proved by Milnor [5] that V' has the homotopy type of a wedge of $2n$-spheres, and the number μ (Milnor number) of $2n$-spheres is readily computable. Recently an interesting formula for μ was given in terms of analytic invariants of a resolution of the singularity at 0 of $f^{-1}(0)$. For surface singularities this is due to Laufer [4]. For higher dimensional singularities this is due to Bennett and the author [1]. In case of two dimensional singularities, various signature formulae are known. For example, Hirzebruch and Mayer [3] have a formula for the signature when f is of the type $x^a + y^b + z^c$ and Steinbrink [6] has a formula for the signature when $f(x, y, z)$ is weighted homogeneous. Recently Durfee [2] has given a formula for the signature σ of V' in terms of topological invariants of a resolution of the singularity at 0 of the complex surface $f^{-1}(0)$. In this paper a formula for the signature of even dimensional singularities is given in terms of topological invariants and analytic invariants of a resolution of the singularity. As a consequence of this formula, a duality theorem for even dimensional strongly pseudoconvex manifolds is proved. It is a pleasure to acknowledge the constant encouragement of Professor Henry Laufer in this research. We would also like to thank Professor Bennett, Professor Coleff, Professor Deligne and Professor Siu for their helpful discussion of mathematics.

2. **Main results.** The ingredient of the proof of Laufer's formula [4] is the Riemann-Roch Theorem. The ingredient of the proof of Durfee's formula (Theorem 1.5 of [2]) is the Hirzebruch index Theorem. We prove the following theorem without using the Riemann-Roch Theorem or the Hirzebruch index Theorem.

THEOREM 1. *Let $f(x, y, z)$ be holomorphic in N, a Stein neighborhood of*

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