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QUOTIENTS OF $C[0, 1]$ WITH SEPARABLE DUAL

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A. Pełczyński [3] and H. P. Rosenthal [5] considered conditions on a bounded linear operator T from $C(K)$, K compact metric, into an arbitrary Banach space which ensure that T is an isomorphism on a subspace of $C(K)$ isomorphic to a $C(S)$ space. (Throughout T will denote a bounded linear operator from $C(K)$, K compact metric, into a Banach space X.) Both used conditions on the "size" of $T^*B_{X^*}$ to produce their results. Pełczyński showed that if T is a nonweakly compact operator there is a subspace Y of $C(K)$ such that Y is isometric to c_0 and the restriction of T to Y is an isomorphism. Rosenthal's result is similar in form: If $T^*B_{X^*}$ is nonseparable then there is a subspace Y of $C(K)$ such that Y is isometric to $C(\Delta)$ (Δ is the Cantor set) and the restriction of T to Y is an isomorphism. Here we announce a similar result for $C_0(\omega^\omega)$, the space of continuous functions on the ordinals not greater than ω^ω and vanishing at ω^ω (the ordinals are considered in the order topology).

To state our result we need to recall the notion of index of a Banach space introduced by Szlenk [6].

DEFINITION. Let A be a bounded subset of a separable Banach space X and B be a bounded w^*-closed subset of X^*. For each $\epsilon > 0$ let $P_0(\epsilon, A, B) = B$ and define inductively a family of subsets of B indexed by the countable ordinals as follows: For each ordinal $\alpha < \omega_1$, $P_{\alpha+1} = \{b \mid \exists (b_n) \subset P_\alpha(\epsilon, A, B),\ b_n \xrightarrow{w^*} b$, and $\exists (a_n) \subset A,\ a_n \xrightarrow{w} 0$, such that $\overline{\lim}_{n\to\infty}\langle b_n, a_n\rangle \geqslant \epsilon\}$ and if β is a limit ordinal, let $P_\beta(\epsilon, A, B) = \bigcap_{\alpha<\beta} P_\alpha(\epsilon, A, B)$. The ϵ-Szlenk index of A and B is

$$\eta(\epsilon, A, B) = \sup\{\alpha: P_\alpha(\epsilon, A, B) \neq \emptyset\}.$$

Before we state our theorem let us note that since T is nonweakly compact if and only if $\eta(\epsilon, B_{C(K)}, T^*B_{X^*}) \geqslant 1$ for some $\epsilon > 0$, we can restate Pełczyński's theorem in terms of the index:

If for some $\epsilon > 0$, $\eta(\epsilon, B_{C(K)}, T^*B_{X^*}) \geqslant 1$, there is a subspace Y of $C(K)$ such that Y is isometric to c_0 and the restriction of T to Y is an isomorphism.

In this form our results are natural extensions of Pełczyński's.

THEOREM 1. *For every $k \in Z^+$ and $\epsilon > 0$ there is an integer $n(\epsilon, k)$ such*

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