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Sums of independent random variables, by V. V. Petrov, Ergebnisse der Mathematik und ihrer Grenzgebiete, Band 82, Springer-Verlag, New York, Heidelberg, Berlin, 1975, x + 345 pp., $39.60.

Many of the early questions in probability theory concern sums of independent random variables, and in the period 1920–1940 research in probability theory was virtually synonymous with the study of sums of independent random variables. It was during these very active 20 years that some of the most exciting theorems were proved. To name a few, which are nowadays standard classroom material: the three series criterion (Khinchine and Kolmogorov, 1925); the equivalence of convergence of an infinite series in distribution, in probability and with probability one (Lévy, 1937); the strong law of large numbers (Kolmogorov, 1933); the law of the iterated logarithm (Khinchine, 1922; Kolmogorov, 1929, Hartman and Wintner, 1941); the classification of the infinitely divisible laws and conditions for convergence to these laws (Lévy, 1934, 1935; Khinchine, 1937); the Lindeberg-Feller theorem (1922, 1935–1937).

With the above results a plateau had been reached in the study of limit theorems, which had started with de Moivre (1730) and Laplace (1812). Already in the late thirties attention started shifting and probabilists became more interested in Markov chains, continuous time processes, and other situations in which the independence between summands no longer applies, and today the center of gravity of probability theory has moved away from sums of independent random variables. Nevertheless, much work on sums of independent random variables continues to be done, partly for their aesthetic appeal and partly for the technical reason that many limit theorems, even for dependent summands, can be reduced to the case of independent summands by means of various tricks. The best known of these tricks is the use of regeneration points, or "Doeblin's trick". In large part the fascination of the subject is due to the fact that applications and the ingenuity of mathematicians continue to give rise to new questions. E.g. renewal theory was inspired by risk theory for insurance companies, the theory of optimal stopping by sequential analysis, invariance principles and functional limit theorems by examples and the search for a simple proof of the Kolmogorov-Smirnov test, fluctuation theory by queueing theory and surprising combinatorial proofs of limit theorems for maxima. Potential theory for random walks, group valued and Banach space valued random walks, imbedding theorems and rates of imitation of normality, the search for "universal laws" (results which hold for partial sums of all sequences of independent identically distributed random variables), the Erdős-Renyi law of large numbers, and sums of random variables indexed by $\mathbf{Z}^d$, $d > 1$, or other graphs, had a more purely mathematical origin.