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EXTENSIONS OF HARDY SPACES AND THEIR USE IN ANALYSIS

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1. Introduction. It is well known that the theory of functions plays an important role in the classical theory of Fourier series. Because of this certain function spaces, the H^p spaces, have been studied extensively in harmonic analysis. When $p > 1$, L^p and H^p are essentially the same; however, when $p \leqslant 1$ the space H^p is much better adapted to problems arising in the theory of Fourier series. We shall examine some of the properties of H^p for $p \leqslant 1$ and describe ways in which these spaces have been characterized recently. These characterizations enable us to extend their definition to a very general setting that will allow us to unify the study of many extensions of classical harmonic analysis.

The theory of H^p spaces on $\mathbf{R}^n$ has recently received an important impetus from the work of C. Fefferman and E. M. Stein [29]. Their work resulted in many applications involving sharp estimates for convolution operators. It is not immediately apparent how much of a role the differential structure of $\mathbf{R}^n$ plays in obtaining these results. Our purpose is to isolate from this theory some of the measure theoretic and geometric properties that enable us to obtain in a unified form many of these applications as well as other results in harmonic analysis. We shall not deal with those questions involving H^p spaces that are not relevant to our purpose. Some general references involving harmonic analysis and H^p spaces are [23], [64], [27], [62], [57] and [55].

The main tool in our development is an extension and a refinement of the Calderón-Zygmund decomposition of a function into a "good" and "bad" part. This tool is presented in the proof of Theorem A and is of a somewhat technical nature. It is included here in order to make the presentation of the theory we develop essentially self-contained. In some examples we give applications of this theory that require material not presented here. We do, however, give the necessary references. In this sense, we hope that this exposition is accessible to a general audience.

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Suppose f is a *real-valued* integrable function on T, the perimeter of the unit disc in the plane (which we identify in the usual way with $[-\pi, \pi)$). Suppose f

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