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TENSOR PRODUCTS OF UNITARY REPRESENTATIONS OF $SL_2(R)$

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1. **Introduction.** We consider the tensor product of two irreducible unitary representations of $G = SL_2(\mathbf{R})$; in particular, we obtain its reduction as a direct integral of irreducible representations. This question has been solved in certain cases by Pukanszky [4] and Martin [3]. We restate their results and also do the remaining cases.

2. **Notation.** Let $M = \{\pm I\}$; $K = SO_2(\mathbf{R})$; and let A (resp. N) be the subgroup consisting of all positive diagonal matrices (resp. upper triangular unipotent matrices). Let

$$h_t = \begin{pmatrix} e^t & 0 \\ 0 & e^{-t} \end{pmatrix} \in A, \qquad k_\theta = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \in K.$$

For $s \in i\mathbf{R}$, $\epsilon \in \hat{M}$, let $\eta_{s,\epsilon}$ be the one-dimensional representation of MAN given by $\eta_{s,\epsilon}$: $mh_tn \longmapsto \epsilon(m) \cdot e^{st}$, $m \in M$, $n \in N$. Let $\pi_{s,\epsilon} = \mathrm{Ind}_{MAN}^G \eta_{s,\epsilon}$, a principal series representation.

For $-1 < \sigma < 0$, let π_σ^c be the (unitary) complementary series representation which is infinitesimally isomorphic to the "nonunitary principal series" representation induced from the representation of MAN given by $mh_tn \longmapsto \exp(\sigma t)$.

The representations π_σ^c are all irreducible, as are all the $\pi_{s,\epsilon}$, except when $s = 0$ and $\epsilon \in \hat{M}$ is nontrivial. In this case, $\pi_{0,\epsilon}$ is the direct sum of two irreducible representations, denoted $\pi_{0,\epsilon}^+$ and $\pi_{0,\epsilon}^-$.

For $n \in \mathbf{Z}$, define $\chi_n \in \hat{K}$ by $\chi_n(k_\theta) = e^{in\theta}$. For $n \geqslant 2$, we let T_n (resp. T_{-n}) be the discrete series representation with lowest weight n (resp. highest weight $-n$). We also let $T_1 = \pi_{0,\epsilon}^+$, $T_{-1} = \pi_{0,\epsilon}^-$, the so-called "mock discrete series representations" with extreme weights 1 and -1 respectively.

The representations we have described exhaust the irreducible unitary representations of G. For details, see, e.g., Lang [2].

3. **A preliminary result.** Before proceeding, we state a very easy but useful fact, for any separable locally compact group.

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