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BORDISM OF DIFFEOMORPHISMS

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1. **Introduction.** In this note we determine the bordism groups Δ_n of orientation preserving diffeomorphisms of n-dimensional closed oriented smooth manifolds. These groups were introduced by W. Browder [1]. Winkelnkemper showed that each diffeomorphism of the sphere S^n is nullbordant [7]. On the other hand, he showed that Δ_{4k+2} is not finitely generated. Medrano generalized this result to Δ_{4k} [5]. For this he introduced a powerful invariant in the Witt group $W_\pm(\mathbf{Z}, \mathbf{Z})$ ($I_\pm$ in Medrano's notation) of isometries of free finite-dimensional $\mathbf{Z}$-modules with a symmetric (antisymmetric) unimodular bilinear form. The invariant is given by the middle homology modulo torsion, the intersection form and the isometry induced by the diffeomorphism. For a diffeomorphism f: $M \longrightarrow M$ we denote this invariant by $I(M, f)$, the isometric structure of (M, f). It is a bordism invariant and leads to a homomorphism I: $\Delta_{2k} \longrightarrow W_{(-1)^k}(\mathbf{Z}, \mathbf{Z})$.

Neumann has shown that the homomorphism I is surjective, that $W_\pm(\mathbf{Z}, \mathbf{Z}) \otimes \mathbf{Q} \cong \mathbf{Q}^\infty$ and that $W_\pm(\mathbf{Z}, \mathbf{Z})$ contains infinitely many summands of orders 2 and 4 [6]. On the other hand, $W_\pm(\mathbf{Z}, \mathbf{Z})$ is a subgroup of $W_\pm(\mathbf{Z}, \mathbf{Q})$, the Witt group of isometries of finite-dimensional $\mathbf{Q}$-vector spaces. This group plays an important role in the computation of bordism groups C_{2k-1} of odd-dimensional knots, which can be embedded in $W_{(-1)^k}(\mathbf{Z}, \mathbf{Q})$. It is known that $W_\pm(\mathbf{Z}, \mathbf{Q}) \cong \mathbf{Z}^\infty \oplus \mathbf{Z}_2^\infty \oplus \mathbf{Z}_4^\infty$ [3]. Thus the group $W_\pm(\mathbf{Z}, \mathbf{Z})$ is also of the form $\mathbf{Z}^\infty \oplus \mathbf{Z}_2^\infty \oplus \mathbf{Z}_4^\infty$.

It turns out that the isometric structure is essentially the only invariant for bordism of diffeomorphisms.

2. **Bordism of odd-dimensional diffeomorphisms.** Two diffeomorphisms (M_1, f_1) and (M_2, f_2) are called bordant if there is a diffeomorphism (N, F) on an oriented manifold with boundary such that $\partial(N, F) = (M_1, f_1) + (-M_2, f_2)$. The bordism classes $[M^n, f]$ form a group under disjoint sum, called Δ_n.

The mapping torus of a diffeomorphism (M, f) is $M_f = I \times M/(0, x) \sim (1, f(x))$. This construction leads to a homomorphism $\Delta_n \longrightarrow \Omega_{n+1}$ ($[M, f] \mapsto [M_f]$), where Ω_{n+1} is the ordinary bordism group of oriented manifolds.

In [4] we proved the following result.

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