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LOCAL GEVREY AND QUASI-ANALYTIC HYPOELLIPTICITY FOR $\Box_b$

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Introduction. The $\bar{\partial}_b$ complex is well defined on any smooth CR manifold M, and once a metric is fixed, so is the complex Laplace-Beltrami operator $\Box_b$ on forms of type (p, q). For compact M without boundary, the $\bar{\partial}_b$ cohomology of M may be studied via $\Box_b$ [6], and thus local smoothness of solutions to $\Box_b u = f$ is important. In its own right, $\Box_b$ is a prototype of doubly characteristic operators. Under suitable convexity conditions on M, Kohn [7] established the following subelliptic estimate on (p, q) forms in $C_0^\infty(M)$:

$$\|\varphi\|_{1/2}^2 \leqslant C(\Box_b\varphi, \varphi) + C'\|\varphi\|_0^2,$$

and in general such estimates imply C^∞ and Gevrey (G^s, $s \geqslant 2$) hypoellipticity locally [3], [8], [10] and no more [1]. In the special case of the Heisenberg group, Folland and Stein [5] found an explicit fundamental solution which gives local analytic hypoellipticity; while, in general, if M is compact, satisfies the convexity condition $Y(q)$ of Kohn, and has an invertible Levi form, the author proved $\Box_b$ is globally analytic hypoelliptic and so is the $\bar{\partial}$-Neumann problem (joint work with M. Derridj, cf. [4], [9]).

In this note we assume $Y(q)$ and the invertibility of the Levi form and prove local regularity in all Gevrey classes G^s with $s > 1$ as well as in a quasi-analytic class. Full details will appear elsewhere.

Notations and definitions. The class $C^L(\Omega) \subset C^\infty(\Omega)$, Ω open in R^n, is defined by the condition that for all $K \subset\subset \Omega$ there exists a constant $C_{f,K}$ such that for any multi-index α,

$$\sup_K |D^\alpha f| \leqslant C_{f,K}^{|\alpha|+1} L(|\alpha|)^{|\alpha|},$$

where we assume that the sequence $\{L(j)\}$ of positive numbers satisfies (1) $L(j)/j$ is nondecreasing and (2) $L(j+1)^{j+1} \leqslant C^j L(j)^j$ uniformly in j. The second condition implies that $C^L(\Omega)$ is closed under differentiation while the first implies that the class is preserved under composition. Thus one may speak of C^L manifolds. If, in addition, $\Sigma L(j)^{-1} < \infty$, the class is called non-quasi-analytic (NQA) and admits compactly supported functions. Common examples are the Gevrey classes $G^s(\Omega)$, obtained by taking $L(j) = j^s$. These are NQA if $s > 1$, while

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