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HALF FACTORIAL DOMAINS[1]

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1. Introduction. Let R be communicative domain with 1. We call R a Half-Factorial Domain (HFD) provided the equality $\Pi_{i=1}^n x_i = \Pi_{j=1}^m y_j$ implies $m = n$, whenever the x's and the y's are nonunit and irreducible elements of R. The study of HFD's is motivated in part by a result in number theory of Carlitz [1], and was raised by Narkiewicz [4]. For a Krull domain R, we denote by $C(R)$ the ideal class group of R. We identify the ideal P with its image in $C(R)$. The ring $\mathbf{Z}$ denotes the ring of integers.

A Dedekind domain R is called special, if whenever a prime P equals (in $C(R)$) a product of primes $Q_1 \cdots Q_t$, it already equals one of the Q's (in $C(R)$).

2. General criteria. Remark that if R is an HFD, then for every $r \neq 0$ if r is a nonunit in R, $w(r) = \{$number of irreducible factors in a complete decomposition for $r\}$ is a well-defined function on $\hat{R} = \{r \mid \neq 0, r \text{ nonunit}, r \in R\}$ into the positive integers, such that $w(rs) = w(r) + w(s)$ for $r, s \in \hat{R}$, and $w(r) = 1$ iff r is an irreducible element in R. Conversely, the existence of such a function implies that R is an HFD.

PROPOSITION 1. *Let R be a Dedekind domain with torsion class group. Then R is an HFD iff whenever $P_1, \ldots, P_t$ are prime ideals so that $P_1^{n_1} \cdots P_t^{n_t} = Rx$, there exists a subproduct $P_1^{m_1} \cdots P_t^{m_t} = Ry$, $m_i \leqslant n_i$, where y is an irreducible element such that $(m_1/s_1) + \cdots + (m_t/s_t) = 1$, where s_i is the order of P_i ($s_i = 1$ if $P_i = Rz$).*

PROPOSITION 2. *Suppose that in the Dedekind domain R, for every prime ideal M there exists a prime ideal N so that MN is a principal ideal. Then R is an HFD iff $C(R)$ is 2-elementary and R is a special Dedekind domain, or else $C(R)$ is isomorphic to Z, and a prime ideal P exists so that for every prime ideal M exactly one of the following equalities holds:* (i) $M \equiv P$, (ii) $MP \equiv R$, (iii) $M \equiv R$.

The following theorem leads to a class of Dedekind domains that are HFD

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