BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 82, Number 4, July 1976

EXISTENCE, UNIQUENESS, STABILITY FOR A SIMPLE FLUID WITH FADING MEMORY

BY MARSHALL SLEMROD

Communicated by R. K. Miller, February 3, 1976

Let Ω be a bounded domain in R^3 with smooth boundary Γ. Let $v_i(\mathrm{x}, t)$ denote the velocity at a point $\mathrm{x} \in \Omega$ at time t of a simple fluid with fading memory when the strain relative to some fixed configuration is small (see [**1**, p. 90]). We assume the fluid is incompressible with density unity. Denote the stress tensor as S_{ij}, δ_{ij} the Kronecker delta, $\cdot$ to be $\partial/\partial t$. Consistent with [**1**] we choose as our constitutive equation

$$S_{ij} + p\delta_{ij} = 2\int_0^\infty m(s)\,[E_{ij}(t-s) - E_{ij}(t)]\,ds \tag{1}$$

where p is an indeterminate pressure, $m(s)$ a material function, E_{ij} the infinitesimal strain tensor. We are considering only a linear theory and must of consistency linearize the basic equation of motion,

$$\dot{v}_i + v_{i,j}v_j = S_{ij,j} \quad \text{in } \Omega, \tag{2}$$

to obtain as a linear model of a simple incompressible fluid with fading memory obeying (1), the equations:

$$\dot{v}_i = -p,_i + \int_0^\infty G(s)v_{i,jj}(t-s)\,ds \quad \text{in } \Omega, \tag{3a}$$

$$v_{j,j} = 0 \quad \text{in } \Omega \text{ (incompressibility)}, \tag{3b}$$

$$v_j = 0 \quad \text{on } \Gamma \text{ (viscous boundary condition)}, \tag{3c}$$

$$v_j(\mathrm{x}, \tau) = v_j^0(\mathrm{x}, \tau), \quad \mathrm{x} \in \Omega, -\infty < \tau \leqslant 0 \tag{3d}$$

$(v_j^0(\mathrm{x}, \tau)$ the initial velocity history).

Here $m(s) = dG(s)/ds$ where $G(s)$ is the shear relaxation modulus, $G(s) \longrightarrow 0$ as $s \longrightarrow \infty$.

Joseph [**2**] has noted that no mathematical theory presently exists for system (3a)–(3d). We have proven a positive existence, uniqueness, stability result in the case

---

AMS (MOS) subject classifications (1970). Primary 76A10, 34J99, 35Q99; Secondary 47D05.

Key words and phrases. Simple fluid, stability, fading memory, contraction semigroup.