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AN EXTENSION OF THE JACOBSON DENSITY THEOREM

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The purpose of this note is to outline a generalization of the Jacobson density theorem and to introduce the associated class of rings.

Throughout R will be an associative ring, not necessarily possessing an identity element. M_R will always denote a right R-module, and homomorphisms will be written on the side opposite to the scalars. For an element $m \in M$, set $(0:m) = \{r \in R \mid mr = 0\}$.

A nontrivial module M_R is called *compressible* if it can be embedded in any of its nonzero submodules. A compressible module M_R is *critically compressible* if it cannot be embedded in any proper factor module. For a compressible module M_R, each of the following conditions is equivalent to M_R being critically compressible: (1) M_R is monoform (i.e. nonzero partial endomorphisms of M are monomorphisms); (2) M_R is uniform and nonzero endomorphisms of M are monomorphisms. The proof of these characterizations is elementary.

In the absence of more suitable terminology let us define a ring to be *weakly primitive* if it possesses a faithful critically compressible module. A weakly primitive ring is prime; for it is easy to see that the annihilator of a compressible module is a prime ideal.

In order to simplify this presentation let us call $(\Delta, {}_\Delta V_R, M_R)$ an *R-lattice* if V is a Δ-R bimodule where Δ is a division ring, $\Delta M = V$, and R acts faithfully on M (so that R can be regarded as a subring of $\mathrm{End}_\Delta V$). We are now prepared to state the main result.

THE DENSITY THEOREM. *The following conditions are equivalent for a ring R.*

(1) *R is weakly primitive.*

(2) *There exists an R-lattice (Δ, V, M) such that given any elements $m_1, \ldots, m_t \in V$ linearly independent over Δ there exists $0 \neq a \in \Delta$ such that for any elements $n_1, \ldots, n_t \in M$ one can find $r \in R$ with $an_i = m_i r \in M$ for each $i = 1, \ldots, t$.*

(3) *There exists an R-lattice (Δ, V, M) such that given any $\tau \in \mathrm{End}_\Delta V$*

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