and have non-p-integral invariants. The proof is due to Serre, and it is supplemented by the author to include also the case where the curves are defined over a function field (of characteristic zero), and their invariants are transcendental. Finally, the field of division points of an elliptic curve A over an algebraic number field K is studied. Let A_0 denote the group of division points of A (= points of finite order), and $K(A_0)$ the field generated by their coordinates over K. If A has no complex multiplication, it is known from Serre's work that the Galois group of $K(A_0)$ over K is an open subgroup of the product $\prod_p \mathrm{GL}_2(Z_p)$ (taken over all primes p). This important theorem is proved here under the additional assumption that the invariant of A is nonintegral.

Part Four enters into the multiplicative theory of elliptic (theta) functions, and the connection to L-series. After first dealing with the analytic theory, exhibiting the classical multiplicative functions and their formulas, the author defines the so-called Siegel functions, which are certain integral modular functions. Their singular values lie in certain well-defined ray class fields; their behavior under Galois automorphisms is deduced from the general reciprocity law of Shimura mentioned above. Two Kronecker limit formulas involving (multiplicative) elliptic functions are established, as well as their relation to L-series over imaginary quadratic number fields. In particular, their value at $s = 1$ is worked out, which plays an important role in algebraic number theory in connection e.g. with class number formulae.

The attentive reader who has travelled up to this point over the ocean of elliptic functions, with this book as his vessel and the author as his guide, will certainly be able to sail further on his own to the scenes of great discoveries past and present. The tour is to be highly recommended even though the sea sometimes may be going rough.

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Maximal orders, by I. Reiner, Academic Press, London, New York, San Francisco, 1975, xii + 395 pp., $36.50.

The theory of orders is a fascinating and difficult subject which occupies much of the common ground between algebra and number theory. Since this theory is known to relatively few contemporary mathematicians, I will give a more than usually thorough survey of the general area, before discussing the book itself.

To facilitate the discussion, we start with the definition and a few examples. Let R be a Dedekind domain (that is, R is a Noetherian integral domain in which all nonzero prime ideals are maximal, and which is integrally closed in its field of fractions K. The last condition means that any element of K which is a zero of a monic polynomial with coefficients in R belongs to R. As examples, one may take principal ideal domains and the rings of algebraic integers of algebraic number fields). An R-order is intended to be a certain type of R-algebra. Some of the examples which should be included are the ring of all $n \times n$ matrices over R (for any $n \geqslant 1$), the group ring RG of a finite