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Self-adjoint operators, by William G. Faris, Lecture Notes in Mathematics, No. 433, Springer-Verlag, Berlin, Heidelberg, New York, 1975, vi+115 pp., $7.80.

Quantum physics has greatly influenced the theory of self-adjoint operators throughout its development and continues to do so today. One problem arising in quantum physics, which is the main problem dealt with in the book under review, is the addition problem: When is the sum of two unbounded self-adjoint operators self-adjoint? More precisely, let A, B with domains $D(A)$, $D(B)$ be self-adjoint operators on a complex Hilbert space H. If the closure C of $A+B$ (defined on $D(A)\cap D(B)$) is self-adjoint, then we can regard C as "the" self-adjoint sum of A and B. More interesting are the cases in which C has many self-adjoint extensions, and the problem is to find the "right" one (if indeed there is a right one).

In quantum mechanics, kinetic and potential energy are described by self-adjoint operators, A, B say. Their sum is the total energy operator C, and to do quantum mechanics one must compute functions of it. One can do this (by the spectral theorem and the associated functional calculus) when C is self-adjoint. In particular, when C is self-adjoint, the dynamics of the system is described by the one parameter unitary group $\{\exp(-itC): t\in \mathbf{R}\}$, which is well defined. An example is the case of a spinless nonrelativistic quantum mechanical particle in a given potential. The Hilbert space is $H=L^2(\mathbf{R}^n)$ and the kinetic and potential energy operators are $A=-\Delta=-\sum_{j=1}^n \partial^2/\partial x_j^2$, $B=$the operator of multiplication by $V(x):\mathbf{R}^n\to\mathbf{R}$ ($B=V(x)$ for short). This set-up also describes two-body problems with no external potentials. The problem is to find the most general conditions on V so that $A+B$ (suitably interpreted) is self-adjoint.

One approach to the addition problem is via the Lie-Trotter product