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ON THE COHOMOLOGY OF H-SPACES OF EXCEPTIONAL TYPE

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Since the 1950's, Araki [1], Borel [3], Bott [4], Cartan, and others investigated homological and homotopical properties of Lie groups. Using the differentiable structure of a Lie group, Araki [1] and Borel [3] calculated the mod p cohomology rings of the exceptional groups. In this note, we indicate that many of their results can be generalized to finite H-spaces. In particular, the proofs of these new results will be completely independent of the existence of an infinitesimal Lie structure.

Let G be a simply connected Lie group. Borel [3] used the classification theorem for simple Lie groups to prove that $H_*(G; Z)$ has no p-torsion for $p \geqslant 7$. The following result follows from a homological argument:

THEOREM 1. *Let X be a simply connected finite H-space. Suppose $H^*(X; Q)$ is isomorphic as algebras to the rational cohomology of an exceptional Lie group. Then $H_*(X; Z)$ has no p-torsion for $p \geqslant 7$ and has 3 or 5 torsion of order at most 3 or 5.*

Theorem 1 is not true in general for finite simply connected H-spaces. John Harper (unpublished) recently discovered finite simply connected H-spaces whose integral homology has p-torsion for any odd prime p.

Let G be a simply connected Lie group. One of the key results used to compute the homology of Lie groups is Bott's result that the homology of the loops on G, $H_*(\Omega G; Z)$, has no torsion. Bott uses Morse Theory [4] to prove this result. We prove the following analogous result for H-spaces:

THEOREM 2 *Let X be a simply connected finite H-space with $H^*(X; Q)$ isomorphic as algebras to the rational cohomology of an exceptional Lie group. Then $H_*(\Omega X; Z)$ has no odd torsion.*

More recently, Hodgkin [5] and Araki [2] proved that for G a simply connected Lie group, $K^*(G; Z)$ is torsion free. Their proofs depend heavily on the differential structure of a Lie group. Hodgkin uses the classification theorem and results of Borel and Araki on the cohomology of Lie groups. Araki uses the existence of a maximal torus.

The following result can be proven using homological methods:

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