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INTEGRAL TRANSFORMS OF WEAK TYPE BETWEEN REARRANGEMENT INVARIANT SPACES

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1. **Introduction.** Let $X(\Omega)$, $Y(\Omega)$ and $Z(\Omega \times \Omega)$ be rearrangement invariant Banach function spaces, where $\Omega = (0, \infty)$ with the Lebesgue measure. Let $M(\Omega)$ be the set of measurable functions on Ω and for every $k \in Z(\Omega \times \Omega)$, denote by z_k the integral operator given by $z_k(f)(x) = \int_\Omega k(x, y)f(y)\,dy$ for $f \in M(\Omega)$, $x \in \Omega$.

In this paper we shall give necessary and sufficient conditions, in terms of the fundamental functions of the spaces (see [2] and [4]) for z_k to be of weak type $\{X, Y\}$ for every $k \in Z$. The methods are similar to those employed by O'Neil in his fundamental paper [3].

2. **The Lorentz $\Lambda(Z)$ and $M(Z)$ spaces.** It is well known how to define the Lorentz Λ and M spaces associated with $X(\Omega)$. To extend these definitions to $Z(\Omega \times \Omega)$, we "smash" Z into $\hat{Z}(\Omega)$, say, via Luxemburg's representation theorem [1]. The relationship between the fundamental functions of these

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