BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 81, Number 3, May 1975

ON THE NORM FORM OF A FINITE GALOIS EXTENSION OVER Q

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Communicated by Robert Fossum, February 13, 1975

1. **Introduction.** Let $\lambda\colon T \longrightarrow T'$ be a $\mathbf{Q}$-isogeny of algebraic tori defined over $\mathbf{Q}$, the rational number field. Then the isogeny λ induces naturally the following maps (cf. [2]):

$$\lambda_v\colon T_v \longrightarrow T'_v, \quad \lambda_v^c\colon T_v^c \longrightarrow T'^c_v, \quad \lambda_{\mathbf{Q}}^\infty\colon T_{\mathbf{Q}}^\infty \longrightarrow T'^\infty_{\mathbf{Q}}, \quad (\hat{\lambda})_{\mathbf{Q}}\colon (\hat{T}')_{\mathbf{Q}} \longrightarrow (\hat{T})_{\mathbf{Q}}.$$

For a homomorphism $\alpha\colon G \longrightarrow G'$ of commutative groups with finite kernel and cokernel, we define the q-symbol of α by $q(\alpha) = [\mathrm{Cok}\ \alpha]/[\mathrm{Ker}\ \alpha]$. Then the q-symbols of the above maps are defined, and $q(\lambda_v^c) = 1$ for almost all finite prime v; more precisely, if K is a finite splitting field for T and T' over $\mathbf{Q}$, then $q(\lambda_v^c) = 1$ whenever v is prime to the degree of λ and is unramified relative to $K/\mathbf{Q}$. In [2], we prove

THEOREM 1. *The relative class number $h_T/h_{T'}$ of T, T' over* $\mathbf{Q}$ *can be expressed as*

$$\frac{h_T}{h_{T'}} = \frac{\tau_T}{\tau_{T'}} \cdot \frac{q(\lambda_\infty)}{q(\lambda_{\mathbf{Q}}^\infty)q((\hat{\lambda})_{\mathbf{Q}})} \cdot \prod_{v \neq \infty} q(\lambda_v^c),$$

where τ_T (resp. $\tau_{T'}$) is the Tamagawa number of T (resp. T') over $\mathbf{Q}$.

In this paper, we apply Theorem 1 to the study of the norm form of a finite Galois extension over $\mathbf{Q}$.

2. **Main theorem.** Let $K/\mathbf{Q}$ be a Galois extension of finite degree n. Denote by N the norm map $R_{K/\mathbf{Q}}(\mathbf{G}_m) \longrightarrow \mathbf{G}_m$, where $\mathbf{G}_m$ is the multiplicative group of the universal domain Ω, and $R_{K/\mathbf{Q}}$ is the Weil functor of restricting the field of definition from K to $\mathbf{Q}$ (cf. [3]). We have an exact sequence

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AMS (MOS) subject classifications (1970). Primary 10C10, 20G30.

[1] This paper is based on a part of the author's Ph.D. thesis, written at Johns Hopkins University under the direction of Professor T. Ono. For the unexplained notions, see [2].