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MONODROMY GROUPS FOR HIGHER-ORDER DIFFERENTIAL EQUATIONS

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ABSTRACT. In this paper we extend certain classical results of Picard and Poincaré (involving monodromy groups of second order equations) to the case of higher-order differential equations.

1. **Preliminaries.** Let F be a compact Riemann surface of genus $g \geqslant 2$ defined by a polynomial equation $P(x, y) = 0$. Use $\xi = (x, y)$ to denote points of F and U to denote the open unit disk. The universal covering map π: $U \to F$ can then be written $\xi = \pi(t)$, where $\pi(t) = (\varphi(t), \psi(t))$. The associated group of cover transformations will be a Fuchsian group Γ satisfying. $F = U/\Gamma$.

Let E_ξ be any finite subset of F. For reasons of simplicity, we shall assume that E_ξ contains all the branch points of F and all the points situated over $x = \infty$. Let E_t be the pre-image of E_ξ under $\xi = \pi(t)$. We consider two basic linear differential equations (with $n \geqslant 2$):

$$\frac{d^n u}{dx^n} + Q_2(x,y)\frac{d^{n-2}u}{dx^{n-2}} + \cdots + Q_n(x,y)u = 0, \qquad (x,y) \in F; \tag{1}$$

$$\frac{d^n v}{dt^n} + q_2(t)\frac{d^{n-2}v}{dt^{n-2}} + \cdots + q_n(t)v = 0, \qquad t \in U. \tag{2}$$

It is understood here that the $Q_\alpha(x, y)$ are rational functions and that the $q_\alpha(t)$ are meromorphic on U. We shall say that (1) is of *type* (E_ξ) iff (1) is of Fuchsian type on F and has all its regular singular points located within E_ξ. See, for example, [2, pp. 123, 478]. Similarly (2) is said to be of *type* (E_t, Γ) iff (2) is an equation of Fuchsian type on U, which has all its singular points contained within E_t and which, in addition, is preserved under the change of variable:

$$(t, v) \to (s, w), \quad s = L(t), \quad w = w(s) = v(t)L'(t)^{(n-1)/2}, \qquad L \in \Gamma. \tag{3}$$

It is important to remark that one can choose branches $\xi_L(t)$ of the

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