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MULTIPLIERS OF CLOSED IDEALS OF $L^p(D^\infty)$

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Let G be a compact abelian group with character group X. Let E be a subset of X and for $1 \leqslant p \leqslant \infty$, let L_E^p be the ideal of E-spectral functions in $L^p(G)$. Let M_E^p be the space of complex-valued functions on E which multiply $\widehat{L_E^p}$ into itself, and let $M^p|_E$ be the set of restrictions to E of functions in M_X^p. We are interested in the following questions:

(i) Does $M_E^p = M^p|_E$?

(ii) Does an analogue of the Riesz-Thorin interpolation theorem hold for the spaces L_E^p? I.e., for $1 \leqslant p_1 \leqslant p_2 \leqslant \infty$, are the interpolation spaces obtained by applying Calderón's complex method of interpolation to $L_E^{p_1}$ and $L_E^{p_2}$ actually the intermediate L_E^p spaces?

(iii) For $1 \leqslant p < q < 2$ or $2 < q < p \leqslant \infty$, is $M_E^p \subseteq M_E^q$?

Question (i) is posed for the circle group $\mathbf{T}$ in [3, pp. 280-281] and has an affirmative answer for any G if $p = 2$ (trivially) or if $p = \infty$ (see [2, Theorem 3.3], [8], and, for a more general result, [6]). Question (ii) is inspired by [1, p. 344, Remarque], while (iii) seems natural in view of (i) and (ii). (An affirmative answer for either (i) or (ii) clearly implies the same for (iii).)

We take for our group G the Cantor group D^∞ $(= Z(2)^{\mathbf{N}})$ and we prove the following:

THEOREM 1. *There exists a subset E of X such that*:

(a) *for each $p \in [1, 2)$, there is a multiplier of $\widehat{L_E^p}$ into $\widehat{L_E^2}$ which is not in $M^p|_E$*;

(b) *the interpolation spaces B_t obtained by applying the complex method of interpolation to L_E^1 and L_E^2 are not the spaces L_E^p $(p = p(t) = 2/(2-t)$, $0 < t < 1)$.*

THEOREM 2. *For $p = 4, 6, \ldots$, there exists a subset $E_p \subseteq X$ such that $M_{E_p}^p \supsetneq M^p|_{E_p}$.*

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