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AN ENTROPY EQUIDISTRIBUTION PROPERTY FOR A MEASURABLE PARTITION UNDER THE ACTION OF AN AMENABLE GROUP

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Throughout this note let G be an arbitrary discrete amenable group. Let $(\Omega, \mathcal{M}, \lambda)$ be a probability space. Let $\mathcal{A}$ be the automorphism group of $(\Omega, \mathcal{M}, \lambda)$. Let T: $G \longrightarrow \mathcal{A}$ be a group homomorphism. We call T an action of G on Ω. For each $g \in G$, let T^g be the image of g in $\mathcal{A}$ under T. Then T^g is a measurable, measure-preserving, invertible map from Ω to itself.

If Q is a partition of Ω and $\omega \in \Omega$, let $Q(\omega)$ be the element of Q which contains ω. If E is a set let $|E|$ denote the cardinality of E.

Let K be a subgroup of G. A net $\{A_\alpha\}$ of finite nonempty subsets of K is said to satisfy property $\mathcal{P}$ with respect to K if $\lim_\alpha |A_\alpha|^{-1} |gA_\alpha \cap A_\alpha| = 1, g \in K$. (Since K is amenable, such a net $\{A_\alpha\}$ exists; see [3].)

Let P be a measurable partition of Ω with finite entropy. If E is a finite nonempty subset of G, let $h_P(E) \in L^1(\Omega)$ be defined as follows:

$$h_P(E)(\omega) = -\log \lambda\left[\left\{\bigvee_{g \in E} (T^g)^{-1} P\right\}(\omega)\right], \qquad \omega \in \Omega.$$

The following generalization of the Shannon-McMillan theorem may be found in [4] and [8]: Let $G = Z^k$, where Z is the group of integers and k is a positive integer. For $n = 1, 2, \cdots$, let $A_n = \{(x_1, x_2, \cdots, x_k) \in Z^k$: $0 \leqslant x_i \leqslant n, i = 1, 2, \cdots, k\}$. Then $\{|A_n|^{-1} h_P(A_n)\}$ converges in $L^1(\Omega)$ as $n \longrightarrow \infty$.

In [7] it is shown that if G is the group of dyadic rationals modulo one, and if A_n is the cyclic subgroup of G generated by 2^{-n}, then $\{|A_n|^{-1} h_P(A_n)\}$ converges in $L^1(\Omega)$ as $n \longrightarrow \infty$. The authors of [7] conjectured that this property generalizes to a general countable abelian group.

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