BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 81, Number 2, March 1975

REAL FORMS OF HERMITIAN SYMMETRIC SPACES[1]

BY HARRIS A. JAFFEE

Communicated by Dock Rim, September 30, 1974

Introduction. The results we give here only begin to answer the following general problems: Let X be a hermitian symmetric domain, Γ a group acting holomorphically and discontinuously, and $U = \Gamma\backslash X$ the quotient. Then, by Kodaira if U is compact and smooth, or, by Baily-Borel if just U has finite volume (and Γ arithmetic), U is algebraic. One can ask for all ways of algebracizing U over $\mathbf{R}$, for each of the number of connected components of $U(\mathbf{R})$, and the type of each component as, say, a real analytic space. For the smooth $\mathbf{R}$-algebraic varieties $U = \Gamma\backslash X$, each component of $U(\mathbf{R})$ is a quotient of the form $U' = \Gamma'\backslash X'$ of a globally symmetric space $X' \subset X$ by a subgroup $\Gamma' \subset \Gamma$. To determine which X' and Γ'-actions occur is our goal.

Generalities. Let X be as above, $\sigma\colon X \to X$ an antiholomorphic involution, and X' the set of fixed points of σ. (We consider Γ only virtually now.)

PROPOSITION. (a) *σ is an isometry of the Bergmann metric.*

(b) *X' is a nonempty connected totally geodesic subsymmetric space of X; $\dim_{\mathbf{R}} X' = \dim_{\mathbf{C}} X$.*

(c) *X' is holomorphically dense: a holomorphic or antiholomorphic automorphism of X is determined by its restriction to X'.*

One can construct, at least for X without "exceptional" factors, (for example via the Lie algebra of the isometry group) involutions as above. Choose σ_0 as "standard" and x_0 a fixed point of σ_0. Let G^h be the group of holomorphic automorphisms of X, K^h the isotropy group at x_0. Then Gal $= \{1, \sigma_0\}$ acts by conjugation on G^h and K^h. Moreover if C denotes the set of all antiholomorphic involutions of X, and C_0 the subset fixing x_0, then G^h acts by conjugation on C and K^h preserves C_0. The quotients C/G^h and C_0/K^h are the G^h- and K^h-conjugacy classes of C and C_0. These can be

AMS (MOS) subject classifications (1970). Primary 32M15, 53C35; Secondary 22E40.
Key words and phrases. Hermitian symmetric domain, antiholomorphic involution.

[1] Partial results of the author's dissertation under M. Kuga at SUNY at Stony Brook.