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ROUND HANDLES AND HOMOTOPY OF NONSINGULAR VECTOR FIELDS

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Introduction. We consider nonsingular vector fields on compact connected C^∞ manifolds. The question is: What orbit structures occur in which homotopy classes of nonsingular vector fields? We show that in dimensions 4 and greater, a nonsingular Morse-Smale (NMS) vector field occurs in each homotopy class. Under the additional assumption that the first Betti number of the manifold is nonzero, we show that a nonsingular volume-preserving (NVP) vector field occurs in each homotopy class. These results are based on the round handle decomposition theorem, interesting in its own right as a structure theorem for manifolds whose Euler characteristic is 0.

Let M be a compact manifold whose boundary has been partitioned into two unions of components: $\partial M = \partial_- M \cup \partial_+ M$, $\partial_- M \cap \partial_+ M = \emptyset$. Then the following are equivalent:

1. $\chi(\partial_- M) = \chi(M)$.
2. $\chi(\partial_+ M) = \chi(M)$.
3. There exists a nonsingular vector field on M pointing inward on $\partial_- M$ and outward on $\partial_+ M$.

DEFINITION. The pair $(M, \partial_- M)$ will be called a *flow manifold* if 1, 2 and 3 above are true. This does not exclude the possibility, of course, that $\partial_- M$, $\partial_+ M$, or ∂M may be empty.

DEFINITION. A nonsingular Morse-Smale (NMS) vector field V on the flow manifold $(M, \partial_- M)$ is one which satisfies (a), (b) and (c) below:

(a) V has nonwandering set equal to a finite number of closed orbits, each having a hyperbolic Poincaré map.

(b) The stable manifold (inset) of one closed orbit is transversal to the unstable manifold (outset) of any other closed orbit.

(c) V points inward on $\partial_- M$ and outward on $\partial_+ M$.

DEFINITIONS. A *round handle of index k* (and dimension n) is a copy

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