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THE CONJUGACY PROBLEM AND CYCLIC AMALGAMATIONS

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Max Dehn first posed the word and conjugacy problems for groups, and solved these problems [2] for the fundamental group G_k for an orientable 2-manifold of genus k. This group has the presentation

$$G_k = (a_1, b_1, \cdots, a_k, b_k; a_1^{-1}b_1^{-1}a_1b_1 \cdots a_k^{-1}b_k^{-1}a_kb_k = 1).$$

We note that G_k is a free product of two free groups with a cyclic amalgamation generated by nonpowers.

The author generalized Dehn's result [3] by solving the conjugacy problem for any free product of free groups with a cyclic amalgamation. On the other hand, Miller [5] gave an example of a free product of two free groups amalgamating finitely generated subgroups which has an unsolvable conjugacy problem. Thus a cyclic amalgamation seems an essential criteria in finding classes of groups with solvable conjugacy problems. (For notational convenience we will speak of a free product "amalgamating u and v" when we mean "amalgamating the cyclic subgroups generated by u and v".)

Anshel and Stebe solved the conjugacy problem [1] for certain *HNN* extensions where the underlying group is free and the extension is obtained by an isomorphism of cyclic subgroups. Following Anshel and Stebe, we say that an element h in a group G is *non-self-conjugate* if its distinct powers are in different conjugacy classes. We will also say that h is *power-solvable* if for any w in G we can decide whether or not w is a power of h. (A group has a *solvable power problem* if all its elements are power-solvable.) We note that every nonidentity element in a free group is non-self-conjugate and power-solvable.

We now are able to state our main result which clearly generalizes Dehn's result.

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