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A COMPLETE LOCAL FACTORIAL RING OF DIMENSION 4 WHICH IS NOT COHEN-MACAULAY

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Samuel [7] stated that he knew of no factorial noetherian ring which was not Cohen-Macaulay. Murthy [6] showed that a geometric factorial ring which is Cohen-Macaulay is Gorenstein. Subsequently, Bertin [1] constructed an example of a factorial ring which was not Cohen-Macaulay. Hochster and Roberts [5] noticed that such examples abound and were found by Serre [9]. On the other hand, Raynaud, Boutot, and Hartshorne and Ogus [3] have shown that a complete local ring which is factorial, of dimension at most 4, and with $\mathbf{C}$ as residue class field is Cohen-Macaulay.

This note is to announce that the completion of Bertin's example (which is characteristic 2) is factorial. This defeats a conjecture suggested by Example 5.9 of Hochster [4] which states: If A is a complete noetherian domain, then some symbolic power of a prime ideal of height one is a maximal Cohen-Macaulay module.

Let k be a perfect field of characteristic p with $p \neq 0$. Let N operate on k^4 by $N(e_i) = e_{i+1}$ for $1 \leqslant i < 4$ and with $N(e_4) = 0$. Then $I + N$ is an automorphism of k^4 of order p if $p \geqslant 5$ and of order 4 if $p = 2$. Let $B = k[X_1, X_2, X_3, X_4]$, which we consider to be the symmetric algebra on k^4. Let G denote the group of automorphisms of B induced by $I + N$. It follows from Samuel [8] that the ring of invariants $A = B^G$ is factorial. If $p = 2$, then Bertin [1] has shown that A is not Cohen-Macaulay. Using a result in Serre [9], Hochster and Roberts [5] show that A is not Cohen-Macaulay if $p \geqslant 5$. Let $S = B_m$ and let $R = S^G$, where $m = (X_1, X_2, X_3, X_4)$. It follows that $R = A_n$ with $n = m \cap A$. The different $D(S/R) = S$, and therefore the cohomology group $H^1(G, \mathbf{G}_m(S)) = 0$. Let $\hat{S}$ denote the m-adic completion of B. The first result is almost obvious.

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