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ON WEAK AND STRONG CONVERGENCE OF POSITIVE CONTRACTIONS IN L_p SPACES

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We consider a linear operator T mapping an L_p space into itself. T will be assumed to be *positive* ($f \geqslant 0 \Rightarrow Tf \geqslant 0$) and a contraction ($\|T\| \leqslant 1$). A matrix (a_{ni}), $n, i = 1, 2, \cdots$, is called *uniformly regular* if

$$(1) \qquad \sup_n \sum_i |a_{ni}| < \infty, \quad \lim_n \sup_i |a_{ni}| = 0, \quad \lim_n \sum_i a_{ni} = 1.$$

THEOREM 1. *If* $1 < p < \infty$ *and if* T *is a positive contraction on* L_p, *then the following conditions are equivalent*

(A) $\lim_n T^n$ *exists in the weak operator topology,*

(B) $\lim_n \Sigma_i a_{ni} T^i$ *exists in the strong operator topology for every uniformly regular matrix* (a_{ni}).

The theorem is already known for $p = 1$ and for $p = 2$, even for not necessarily *positive* contractions ([2], [5]).

SKETCH OF PROOF. (i) The implication (B) $\Rightarrow$ (A) is easy and holds in more general situations (cf. [5]). Hence we only prove (A) $\Rightarrow$ (B).

(ii) If G is the largest set such that G supports a T-invariant function g, then $f \in L_p(G)$ implies that $Tf \in L_p(G)$. $Tg = g$ implies $T^* g^{p-1} = g^{p-1}$, hence $f \in L_{p/(p-1)}(G)$ implies $T^* f \in L_{p/(p-1)}(G)$. Therefore there is no communication between G and $F = G^c$, and the restrictions of T to $L_p(G)$ and $L_p(F)$ may be considered separately. On G there exists a strictly positive T-invariant function, and therefore the theorem for $L_p(G)$ follows from the results proved in [5, §2]. There exist no nontrivial positive T-invariant functions on F, and hence the weak limit of T^n restricted to $L_p(F)$ must be zero.

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