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MAXIMAL MONOTONE OPERATORS IN NONREFLEXIVE BANACH SPACES AND NONLINEAR INTEGRAL EQUATIONS OF HAMMERSTEIN TYPE[1]

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Let Y be a Banach space, Y^* its conjugate space, X a weak*-dense closed subspace of Y^* with the induced norm. We denote the pairing between x in X and y in Y by (y, x). If T is a mapping from X into 2^Y, T is said to be monotone if for each pair of elements $[x, y]$ and $[u, w]$ of $G(T)$, the graph of T, we have $(y-w, x-u)\geqq 0$. T is said to be maximal monotone from X to 2^Y if T is monotone and maximal among monotone mappings in the sense of inclusion of graphs.

The theory of maximal monotone mappings has been intensively developed in the case in which Y is reflexive and $X=Y^*$. In this note, we present an extension of this theory to the case in which X and Y are not reflexive, and show that this extended theory can be used to give a new and more conceptual proof of a general existence theorem for solutions of nonlinear integral equations of Hammerstein type established by the writers in **[2]** by more concrete arguments.

An essential tool in our discussion is supplied by the following definitions:

DEFINITION 1. *Let T be a mapping from X into 2^Y. Then T is said to be X-coercive if for each real number k, the set $\{x \mid x \in X$, there exists w in $T(x)$ such that $(w, x)\leqq k\|x\|\}$ is contained in a convex weak* compact subset A_k of X.*

THEOREM 1. *Let T be a monotone mapping from X to 2^Y. Suppose that 0 lies in $D(T)$, the effective domain of T, and that T is X-coercive. Then the range $R(T)$ of T is all of Y.*

We use the following extension of the concept of pseudo-monotonicity **[1]**:

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