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CHARACTERIZATIONS OF KNOTS AND LINKS

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Although there are inequivalent classical knots with isomorphic groups, J. Simon recently characterized each knot type by a group: the free product of two, suitably chosen, cable-knot groups [4]. In this paper, we announce other characterizations, both algebraic and geometric, that are more direct, cover links as well as knots, and yield characterizations of amphicheiral knots.

In Section 1, we give preliminaries and state two lemmas. In Section 2, we outline the proof of the new characterizations, the combined results of the papers [7] and [8], which contain detailed proofs.

1. **Preliminaries.** Throughout this work, the three-sphere S^3 has a fixed orientation; all maps are piecewise linear; all submanifolds, subpolyhedra; and all regular neighborhoods, at least second regular. If L is a link in S^3, then $\{L\}$ denotes the (ambient) isotopy type of L; the symbol L^*, the mirror image of L.

Let $L\ (= K_1 \cup \cdots \cup K_\mu)$ be a link in S^3. For each of $i = 1, \cdots, \mu$, let V_i be a closed regular neighborhood of K_i and let $\mathcal{K}_i$ be a knot in $\operatorname{Int} V_i$. We assume that $V_i \cap V_j = \varnothing$ when $i \neq j$. We also assume that V_i has order greater than zero with respect to $\mathcal{K}_i$ $(i = 1, \cdots, \mu)$. We set $R(L) = \mathcal{K}_1 \cup \cdots \cup \mathcal{K}_\mu$, and we call $R(L)$ a revision of L.

Let (ρ, η) be a pair of integers; ρ, arbitrary; $\eta = \pm 2$. For each of $i = 1, \cdots, \mu$, let Y_i denote a singular disk that has exactly one clasping singularity, that belongs to $\operatorname{Int} V_i$, and that has ρ as its twisting number, η as its intersection number with its boundary, and K_i as its diagonal; see [3, Section 20, p. 232]. If $\mathcal{K}_i$ is the boundary of Y_i, we shall denote $R(L)$ by $D(L; \rho, \eta)$ and call it the (ρ, η)-double of L. Note that $D(K_i; \rho, \eta)\ (= \mathcal{K}_i)$ is the (ρ, η)-double of K_i.

LEMMA 1.1. *Let* $L(= K_1 \cup \cdots \cup K_\mu)$ *be a link in* S^3, *and let* $R(L)$ *be any revision of* L. *Then* L *is splittable if and only if* $R(L)$ *is splittable.*

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