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A FIXED POINT THEOREM FOR MULTIVALUED NONEXPANSIVE MAPPINGS IN A UNIFORMLY CONVEX BANACH SPACE

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Let C be a nonempty weakly compact convex subset of a Banach space X, and $\mathscr{C}(C)$ be the family of nonempty compact subsets of C equipped with the Hausdorff metric. Let $T:C\to\mathscr{C}(C)$ be a nonexpansive mapping, i.e. for each $x, y \in C$,

$$H(T(x), T(y)) \leqq \|x - y\|,$$

where $H(A, B)$ denotes the Hausdorff distance between A and B. A point $x \in C$ is called a fixed point of T if $x \in Tx$. Fixed point theorems for such mappings T have been established by Markin **[11]** for Hilbert spaces, by Browder [2] for spaces having weakly continuous duality mapping, and by Lami Dozo [7] for spaces satisfying Opial's condition. Lami Dozo's result is also generalized by Assad and Kirk **[1]**. By making use of Edelstein's asymptotic center [4], [5], we are able to prove Theorem 1. Let C be a closed convex subset of a uniformly convex Banach space and let $\{u_i\}$ be a bounded sequence in C. The asymptotic center x of $\{u_i\}$ in (or with respect to) C is the unique point in C such that

$$\limsup_i \|x - u_i\| = \inf\left\{\limsup_i \|y - u_i\| : y \in C\right\}.$$

The number $r=\inf\{\limsup_i\|y-u_i\| : y \in C\}$ is called the asymptotic radius of $\{u_i\}$ in C. Existence of the unique asymptotic center is proved by Edelstein in [5]. Results on ordinal numbers used here may be found in **[13]**.

THEOREM 1. *Let X be a uniformly convex Banach space and C be a closed convex bounded nonempty subset of X. Let $T:C\to\mathscr{C}(C)$ be a nonexpansive mapping from C into the family of nonempty compact subsets of*

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