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A BOUNDARY MAXIMUM PRINCIPLE FOR DEGENERATE ELLIPTIC-PARABOLIC INEQUALITIES, FOR CHARACTERISTIC BOUNDARY POINTS

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Let

$$Lu = \sum_{i,j=1}^{n} a_{ij}(x) \frac{\partial^2 u}{\partial x_i \partial x_j} + \sum_{j=1}^{n} b_i(x) \frac{\partial u}{\partial x_i}, \qquad a = (a_{ij}(x)) \geqq 0,$$

be defined on an open set $\Omega \subset R^n$. We consider solutions $u=u(x) \in C^2(\Omega) \cap C(\bar{\Omega})$ of $Lu \geqq 0$ which attain their maximum value M at P, a characteristic boundary point of Ω.

PROPAGATION SET. (See [3] and [1].) Let the diffusion vector field $\alpha^k(x)$ be the kth column vector of the $n \times n$ matrix $\alpha(x)$, where $\alpha^2=a$, and let the drift vector field $\beta(x)$ be defined by $\beta_i=b_i-\sum_{j=1}^n (a_{ij})_{x_j}$, $i=1, \cdots, n$. Assume that $\alpha^1, \alpha^2, \cdots, \alpha^n, \beta \in C^1(B)$, where $B \subseteq R^n$ is open and $\bar{\Omega} \subseteq B$. For $P \in \delta\Omega$, the propagation set $S(P, \Omega)$ is generated by segments of trajectories in Ω of vector fields $\lambda_0\beta+\lambda_1\alpha^1+\lambda_2\alpha^2+\cdots+\lambda_n\alpha^n$, $\lambda_0 \geqq 0$, where the scalar functions $\lambda_k=\lambda_k(x) \in C^1$, $k=0, 1, \cdots, n$. $\bar{S}(P, \Omega)$ is the closure of $S(P, \Omega)$ in Ω.

CURVATURE CONDITION. Let $P \in \delta\Omega$ be a characteristic boundary point, that is, $\nu a(P)\nu=0$, where ν is the unit inner normal to $\delta\Omega$ at P. Then $\nu\alpha^k(P)=0$ for each k, $k=1, \cdots, n$. Let $\prod^k$ be the plane of $\alpha^k(P)$ and ν through P. In this plane, the cross-section of $\delta\Omega$ and the projection of the trajectory of α^k through P are curves which are perpendicular to ν at P. Let the curvatures of these curves at P be τ_k, for the sectional curvature of $\delta\Omega$, and σ_k, for the 'shadow curvature' [4] of the trajectory of α^k. Finally, define the 'excess curvature' ρ_k to be the difference $\tau_k-\sigma_k$.

LEMMA. *$\beta\nu-\sum_{k=1}^n \rho_k|\alpha^k|^2=b\nu-\sum_{k=1}^n \tau_k|\alpha^k|^2$ at P. (The sums are over those k for which $\alpha^k(P)\neq 0$.)*

For the following results to hold, the curvature condition $\beta\nu-\sum_{k=1}^n \rho_k|\alpha^k|^2>0$ must be satisfied at P.

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[1] This note summarizes results contained in the author's Ph.D. thesis, which was written under the direction of Professor Raymond Redheffer at the University of California, Los Angeles.