BULLETIN OF THE
AMERICAN MATHEMATICAL SOCIETY
Volume 80, Number 2, March 1974

WEAKLY ALMOST PERIODIC FUNCTIONS WITH ZERO MEAN

BY CHING CHOU[1]

Communicated by Kenneth A. Ross, August 30, 1973

Let G be a locally compact group, $C(G)$ the space of bounded complex-valued continuous functions on G with the sup norm and $UC(G)$ the closed subspace of $C(G)$ consisting of uniformly continuous functions. If $f \in C(G)$ and $x \in G$ the left {right} translation of f by x, $l_x f$ $\{r_x f\}$, is given by $(l_x f)(y)=f(xy)$ $\{(r_x f)(y)=f(yx)\}$. $f \in C(G)$ is said to be weakly almost periodic (w.a.p.) if the set $\{l_x f : x \in G\}$ is relatively compact with respect to the weak topology of $C(G)$. The set of all w.a.p. functions on G, designated by $W(G)$, is a closed subalgebra of $UC(G)$ and is closed under translations. Moreover, it has a unique invariant mean $m : m \in W(G)^*$, $\|m\|=1$, $m \geqq 0$ and $m(l_x f)=m(r_x f)=m(f)$ for $x \in G$ and $f \in W(G)$. (This is the well-known Ryll-Nardzewski fixed point theorem. Burckel's monograph [**1**] is a convenient reference for this and other facts concerning w.a.p. functions.) Let $W_0(G)=\{f \in W(G) : m(|f|)=0\}$. Then $W_0(G)$ is a closed subalgebra of $W(G)$ and $W(G)=W_0(G) \oplus A(G)$ where $A(G)= \{f \in C(G) : f \text{ almost periodic}\}$, cf. [**1**]. Therefore the study of $W(G)$ is somehow reduced to the study of $A(G)$ and $W_0(G)$. For almost periodic functions there are useful characterization theorems and a deep approximation theorem; cf. [**4**]. But little is known about functions in $W_0(G)$. One well-known fact is that $C_0(G) \subset W_0(G)$ if G is noncompact. Here $C_0(G)=\{f \in C(G) : f \text{ vanishing at infinity}\}$. Burckel [**1**, Theorem 4.17] proved that if G is abelian and noncompact then $C_0(G) \subsetneqq W_0(G)$. His proof is short but some quite deep results in harmonic analysis on abelian groups are used. He conjectured that the abelian hypothesis is inessential. Even though the abelian hypothesis can be weakened considerably, there exists a noncompact solvable group G with $C_0(G)=W_0(G)$.

Let $\boldsymbol{C}$ be the set of complex numbers, $\boldsymbol{T}$ the unit circle in $\boldsymbol{C}$ and $\boldsymbol{R}$ the set of real numbers.

AMS (MOS) subject classifications (1970). Primary 22A10, 43A07, 46N05; Secondary 22D99, 43A60, 46E15.

[1] Supported in part by NSF grant PO 34190-001.