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THE CALABI CONSTRUCTION FOR COMPACT RICCI FLAT RIEMANNIAN MANIFOLDS

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1. **The main result and some consequences.** In 1956 E. Calabi [**6**] attacked the classification problem of compact euclidean space forms by means of a special construction, called the Calabi construction (see Wolf [**14**, p. 124]). Here we announce that the construction can be extended to compact riemannian manifolds whose Ricci curvature tensor is zero (Ricci flat). Of course, it is not known if there exist any Ricci flat nonflat compact riemannian manifolds, and in fact a search for such manifolds was the original motivation for our study. However, as a consequence of our extension of Calabi's result we reduce the question of existence of a compact nonflat Ricci flat manifold to the simply connected, connected case. In any case, we essentially reduce the construction of compact Ricci flat manifolds to the lower-dimensional case together with the case of first Betti number zero.

As a further consequence of our construction we extend one of the Bieberbach theorems [**4**], [**14**, Theorem 3.3.1] from the flat to the Ricci flat case (Theorem 1.4) and give various sufficient topological conditions for a Ricci flat manifold to be flat.

Our main result is the following:

MAIN THEOREM 1.1. *Let M^n be a compact connected Ricci flat riemannian n-manifold with first Betti number $k=b_1(M^n)$. Then there is a finite normal riemannian covering*

$$p: T^k \times M^{n-k} \to M^n = \Psi \backslash T^k \times M^{n-k}$$

where

(1) *T^k is a flat riemannian k-torus;*

(2) *$\Psi=\{(h(\varphi), \varphi) | \varphi \in \Phi\}$ where Φ is a finite group of isometries of M^{n-k}, h is an injective homomorphism of Φ into the translation group of T^k (so Ψ acts freely and properly discontinuously on $T^k \times M^{n-k}$);*

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