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THE SUBSCRIPT OF $\aleph_n$, PROJECTIVE DIMENSION, AND THE VANISHING OF $\varprojlim^{(n)}$

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In this sketch, I will try to indicate how cardinality questions and manipulations became intimately connected with homological algebra. The modern form of this subject stems from the book *Homological algebra*, by H. Cartan and S. Eilenberg, published in 1956. What began as a study of dimension via derived functors branched off into a study of dimension via cardinality and came back to a study of derived functors via cardinality. I will give an historical sketch of this. All rings are associative with 1, all modules are unital right modules unless otherwise stated.

1. **Projective dimension and** Ext. Let us begin by defining the main topic of concern, projective dimension. Let M be an R-module. Then we can express M as a quotient of a free R-module; for example take the obvious epimorphism from $P_0 = \bigoplus_{x \in M} R_x$ to M. One gets a short exact sequence of modules

$$0 \to K_1 \to P_0 \to M \to 0.$$

If one defines an equivalence relation on the category of right R-modules by $A \sim B$ if and only if there exist free R-modules P and P' such that $A \oplus P \approx B \oplus P'$, then the equivalence class of K_1 depends only on the equivalence class of M. Now iterate this procedure to get a family of short exact sequences

$$(*) \qquad \begin{array}{c} 0 \to K_1 \to P_0 \to M \to 0 \\ 0 \to K_2 \to P_1 \to K_1 \to 0 \\ \vdots \\ 0 \to K_n \to P_{n-1} \to K_{n-1} \to 0 \\ \vdots \end{array}$$

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