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UNLINKING UP TO COBORDISM

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0. **Introduction.** Let S_i be a copy of S^n ($n \geqq 3$) for $i = l, \ldots, m$. An m-link of dimension n is an embedding $L: S_1 + \cdots + S_m \to S^{n+2}$, where $+$ stands for disjoint union.

We say L is a boundary link if it extends to an embedding $V_1 + \cdots + V_m \to S^{n+2}$, where V_i is a $(n + 1)$-dimensional, compact, framed manifold with $\partial V_i = S_i$. The V_i are called *Seifert* manifolds for L. In particular, if the V_i are disks, we say that L is trivial.

A link L is split if we can find $(n + 1)$-spheres Σ_j ($j = 1, \ldots, m - 1$), smoothly embedded in S^{n+2} and disjoint from $\operatorname{Im}(L)$ as well as from each other, and such that each of the m connected components of $S^{n+2} - \bigcup \Sigma_j$ contains one of the knots $L(S_i)$.

In [**2**], the notion of cobordism is defined. The cobordism classes of m-links of dimension n form, under componentwise connected sum, an abelian group $C_n^{(m)}$. The group $C_n = C_n^{(1)}$ has been computed in [**4**] for n odd $\geqq 3$, and found to be trivial for n even $\geqq 2$ in [**2**]. The purpose of this note is to announce the following result:

Every m-link of dimension $n \geqq 3$ is cobordant to a split link; in particular:
If n is odd $\geqq 3$, $C_n^{(m)} = C_n \oplus \cdots \oplus C_n$ (m times).
If n is even $\geqq 4$, $C_n^{(m)} = 0$.

1. **The fundamental group.** The normal bundle of $\operatorname{Im}(L) \subset S^{n+2}$ is trivial; let

$$X = \overline{S^{n+2} - (T_1 + \cdots + T_m)}$$

where T_i is a tubular neighborhood of $L(S_i)$ diffeomorphic to $S_i \times D^2$. The compact manifold X is called the *complement* of L and $\pi = \pi_1(X)$ its *group*. Observe, $\partial X = (S_1 \times S^1) + \cdots + (S_m \times S^1)$.

The inclusion $\partial X \subset X$ induces a homomorphism of fundamental groups $h: F_m \to \pi$, where F_m is the free group in m generators a_i. The elements $h(a_i)$ are called *meridians* of L.

LEMMA 1. *The homomorphism h induces a monomorphism*

$$h_*: F_m \to \pi/\pi_\omega$$

where π_ω is the ωth term of the lower central series of π (cf. [**6**, *p.* 157]).

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