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SOME POLYNOMIAL ALGEBRAS OVER THE STEENROD ALGEBRA A_p

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The problem of classifying the possible cohomology algebras of topological spaces has usually been attacked negatively by introducing cohomology operations and requiring these to be compatible with the structure of the candidate for a cohomology algebra. Even in the special case that the $\boldsymbol{Z}/p$ cohomology is a polynomial algebra, the information obtained in this way is limited. For example, Steenrod [3] suggested a method of verifying that a given Z/p polynomial algebra is a module over the Steenrod algebra A_p and carried out the computations for

$$\{(\boldsymbol{Z}/p)[x_4, y_{4+2(p-1)}], P_p^1 x = y\}.$$

Such an approach has the drawbacks of being tedious to apply and also not producing a space with the given cohomology algebra. In this note, some ideas from Lie group theory and a generalization of a construction of Sullivan [4] are used to expand the rather-small list of known polynomial cohomology algebras. In particular, the example treated by Steenrod is realized as the $\boldsymbol{Z}/p$ cohomology of a space. In a later paper [5] the analogy of this construction with some properties of the Lie groups will be pursued and an application to the construction of some new mod p loop spaces given.

NOTATION. $\boldsymbol{Z}/p = \{\text{integers mod } p\}$,
$\hat{\boldsymbol{Z}}_p = \{p\text{-adic integers}\}$,
$\boldsymbol{Q}_p = \{p\text{-adic numbers}\}$,
$K(\hat{\boldsymbol{Z}}_p, 2) =$ Eilenberg-Mac Lane space of type $(\hat{\boldsymbol{Z}}_p, 2)$.

LEMMA 1.1. *$H^*(K(\hat{\boldsymbol{Z}}_p, 2), \boldsymbol{Z}/p) = (\boldsymbol{Z}/p)[x]$, $\dim x = 2$.*

DEFINITION 1.2 [2]. Let W be a finite subgroup of $GL(\boldsymbol{Q}_p, n)$. W is *generated by reflections* in $\boldsymbol{Q}_p^n$ if W is generated by elements which have a hyperplane pointwise invariant. These elements thus have $n - 1$ eigenvalues of $+1$ and the remaining eigenvalue an rth root of unity (r must of course divide $p - 1$). Also, since W is finite, we may consider it to be in $GL(\hat{\boldsymbol{Z}}_p, n)$.

Construction 1.3. Let W be generated by reflections in $\boldsymbol{Q}_p^n$. By choosing an action of W on $\pi_2(K(\bigoplus_n \hat{\boldsymbol{Z}}_p, 2))$, W may be given an action on

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