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ABSOLUTELY SUMMING, L_1 FACTORIZABLE OPERATORS AND THEIR APPLICATIONS

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Grothendieck asked in [3], Problem 2, p. 72 and in the remarks on p. 39, whether every 1-absolutely summing operator between two Banach spaces can be factored through an L_1 space. Theorem 2 announces the negative answer to this question.

Corollaries 1 and 2 provide counterexamples to two other questions equivalent to Problem 2, and mentioned in [3]: Can every operator T, whose adjoint T' is 1-absolutely summing, be factored through a $C(K)$ space? Is every operator which has the form UV, where U' and V are both 1-absolutely summing, an integral operator?

Theorems 3 and 5 establish the existence of a sequence of finite-dimensional Banach spaces which have the property that their unconditional basis constants tend to infinity. This answers the question mentioned for example in [1], [4], [5], [7] and asked also by A. Pełczyński and H. P. Rosenthal.

Part (5) of Theorem 4 settles a conjecture of McCarthy [8, p. 269] regarding the distance of $\mathscr{L}(l_2^n, l_2^n)$ from the subspaces of l_1.

Theorem 5 answers Problem 2 [6] by proving that when $1 \leqq p \neq 2 \leqq \infty$, M_{σ_p} (see definition below) has no unconditional basis.

Detailed proofs of these and other results will be given elsewhere.

Let $\mathscr{L}(E, F)$ denote the space of operators between two Banach spaces E and F. $E \otimes^\alpha F$ denotes the completion under the α norm of the algebraic tensor product $E \otimes F$. In particular, $l_2 \otimes^\vee l_2$, $l_2 \otimes^\wedge l_2$ and $l_2 \otimes^\sigma l_2$ are the spaces of *compact*, *integral* and *Hilbert-Schmidt* operators respectively, from l_2 to l_2. Here $\vee$ and $\wedge$ denote the "least" and "greatest" cross-norms respectively [3]. Other classes of operators considered here are:

(1) $\Pi_p(E, F)$ $(1 \leqq p \leqq \infty)$, the space of *p-absolutely summing* operators from E to F equipped with the norm π_p [9].

(2) $I_p(E, F)$, the space of *p-integral* operators from E to F equipped with the norm i_p [10].

(3) $\Gamma_p(E, F)$, the space of *L_p-factorizable* operators from E to F, that is, $T \in \Gamma_p(E, F)$ if and only if $T \in \mathscr{L}(E, F)$ and there is a positive measure space (Ω, Σ, μ) and $A \in \mathscr{L}(E, L_p(\mu))$, $B \in \mathscr{L}(L_p(\mu), F'')$ such that $iT = BA$,

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